

Chapter 1

Introduction

The field of metamaterials arose as a combination of advances in fabrication capabilities and developments in the physical understanding of how matter interacts with electromagnetic waves. Metamaterial advances span the electromagnetic spectrum, with examples being more common at lower (e.g., microwave) frequencies. The microwave or x-band regime has proven to be a good testbed for the first generation of metamaterials; however, recently, we have seen optical-range metamaterials emerging as well. The discovery of these more complex material–wave interactions has come about when nanofabrication techniques allow structures to be formed with dimensions much smaller than the wavelength of (e.g., visible) light. We can now, in principle, engineer material properties as we see fit, through arrangements of meta-atomic structures, or small geometrically designed collections of atoms that have very specific responses to incident radiation. Losses in these materials will remain a major challenge, as we will see.

1.1 Historical Perspective

Most of the research in engineering and physics during the latter part of the 20th century was performed by researchers who had no knowledge of Victor Vesalago’s 1968 paper³ predicting some unusual material properties. These properties included refraction on the same side of the normal for a material that had a negative value for both its permittivity and its permeability. It was not until Pendry’s 2000¹¹ paper pointed out some applications of this type of material that interest in making a material with these characteristics took off. Pendry proposed that the normally exponentially decaying components of a wave, namely, the evanescent components (see Chapter 6), might be amplified by this type of material. This possibility led to the promise of a ‘perfect’ lens, or a lens that would have no theoretical resolution limit, and this generated enormous enthusiasm to study these new ‘metamaterials.’

How do we make a metamaterial? We first look at the descriptions of simple bulk material properties and see how these can then be applied to metamaterial constituents.

1.2 Basic Electromagnetic Properties of Materials

Originally, material interaction with electromagnetic waves was studied only in the visible spectrum because of the human detector. Once the relationships between the electromotive force and fields were settled, pioneers like Heinrich Hertz quickly expanded their research into other frequency ranges. In the wider realm of electromagnetic radiation, it became clear that both visible light and these lower-frequency waves were all part of a continuum. Traditionally, the index of refraction n has served as the simple measure by which to evaluate the optical properties of a material in terms of how much one can bend a light beam. To understand the index of refraction we must have a basis from which to describe material interaction with light, namely, Maxwell's equations.

1.3 Maxwell's Equations

We expect the reader to be somewhat familiar with Maxwell's equations, thus we do not fully derive them here as we are primarily concerned with their consequences. We first follow the historical development and explanations of Maxwell's equations. Afterwards, we outline some of the requirements derived from Maxwell's equations. Finally, we consider their physical meaning and then how these equations are modified in the presence of matter. The models for the electromagnetic properties of matter are also reviewed as a foundation for understanding wave interactions with atoms, real materials, and engineered material comprising meta-atoms, i.e., metamaterials.

Although we might think that progress was slow, we should remind ourselves that it was not until the 'voltaic pile' around 1800 that a consistent source of current was available. Not long after, in 1820 while using this type of battery, Hans Christian Oersted noticed that a nearby compass needle was deflected by a parallel wire. Almost simultaneously, André-Marie Ampère presented a detailed study of this phenomena that relates the closed path integral of the magnetic field to the electric current density, which can be formalized to a form similar to³

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 \int \mathbf{J} \cdot \mathbf{n} dS. \quad (1.1)$$

This led to the obvious hypothesis that magnetism could, in some way, also create an electric response. Accomplishing this proved more difficult, as sources of magnetic current are limited, to say the least, even if there were magnetic monopoles. Faraday was performing such an experiment when he

recognized that by moving a magnet within a wire loop, he was able to create a current. Thus, the changing magnetic flux was causing an induced electrical current, which can be determined by the electromotive force,

$$\mathcal{E}_{mf} = \left| \frac{\partial \Phi_B}{\partial t} \right|. \quad (1.2)$$

Of course, these formalisms are missing a few key components, and Faraday realized that there was a more complicated law of induction at work. The idea of flux was carried further by Carl Friedrich Gauss to relate the electric flux through a closed surface due to electric charge, q_{enc} :

$$\Phi_E = \frac{q_{enc}}{\epsilon_0}, \quad (1.3)$$

which, as far as we know, can be extended to magnetism using the same principle of superposition with the assumption that there are no magnetic monopoles:

$$\Phi_B = 0. \quad (1.4)$$

It was a considerable feat to realize that the relationship among these equations can be combined to formulate the fundamentals of electromagnetics. Although we have to note that Maxwell initially started with over 20 equations to relate these, it was in fact Oliver Heaviside who reduced them further to the four equations now known as Maxwell's equations.⁴

1.4 Differential Form of Maxwell's Equations

We now state the differential formulation of Maxwell's equations, which we intend to use for the remaining derivations within this text:

$$\nabla \cdot \mathbf{D}(\mathbf{r}, t) = \rho(\mathbf{r}, t) \quad (1.5a)$$

$$\nabla \cdot \mathbf{B}(\mathbf{r}, t) = 0 \quad (1.5b)$$

$$\nabla \times \mathbf{H}(\mathbf{r}, t) = \frac{\partial}{\partial t} \mathbf{D}(\mathbf{r}, t) + \mathbf{J}_e(\mathbf{r}, t) \quad (1.5c)$$

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\frac{\partial}{\partial t} \mathbf{B}(\mathbf{r}, t), \quad (1.5d)$$

where

$\mathbf{E}(\mathbf{r},t)$ = electric field strength (volts/meter)

$\mathbf{B}(\mathbf{r},t)$ = magnetic flux density (webers/meter²)

$\mathbf{D}(\mathbf{r},t)$ = electric flux density (coulombs/meter²)

$\mathbf{H}(\mathbf{r},t)$ = magnetic field strength (amperes/meter)

$\mathbf{J}_e(\mathbf{r},t)$ = electric current density (amperes/meter²)

$\rho(\mathbf{r},t)$ = charge density (coulombs/meter³).

We should also note that this requires the addition of two more equations to formally complete fundamental derivations:⁵

$$\mathbf{D}(\mathbf{r},t) = \epsilon_0 \mathbf{E}(\mathbf{r},t) + \mathbf{P}(\mathbf{r},t) = \epsilon_0 \epsilon_r \mathbf{E}(\mathbf{r},t) = \epsilon \mathbf{E}(\mathbf{r},t) \quad (1.6)$$

$$\mathbf{B}(\mathbf{r},t) = \mu_0 \mathbf{H}(\mathbf{r},t) + \mathbf{M}(\mathbf{r},t) = \mu_0 \mu_r \mathbf{H}(\mathbf{r},t) = \mu \mathbf{H}(\mathbf{r},t), \quad (1.7)$$

where³

$\mathbf{P}(\mathbf{r},t)$ = electric polarization vector (coulombs/meter²)

$\mathbf{M}(\mathbf{r},t)$ = magnetic polarization vector (webers/meter²)

ϵ = permittivity $\frac{(\text{amperes} \times \text{seconds})}{(\text{volts} \times \text{meters})}$

ϵ_0 = permittivity of free space $8.8541878 \times 10^{-12} \frac{(\text{amperes} \times \text{seconds})}{(\text{volts} \times \text{meters})}$

ϵ_r = relative permittivity dimensionless

μ = permeability $\frac{(\text{volts} \times \text{seconds})}{(\text{amperes} \times \text{meters})}$

μ_0 = permeability of free space $1.25663706 \times 10^{-6} \frac{(\text{volts} \times \text{seconds})}{(\text{amperes} \times \text{meters})}$

μ_r = relative permeability dimensionless

The current formulation and understanding of these physical quantities requires the duality of the wave and particle nature of light, although many of the derivations are still based on classical approximations. It was Hendrik Lorentz who removed the barrier in his 1892 theory of electrons by separating mechanical qualities from those of light, thus effectively paving the way for the separation of electromagnetic radiation and its interaction with normal matter.⁷

With the four equations (1.5) and a few additional constitutive relationships, we describe the material properties that we will exploit in order to develop the interesting properties associated with engineered materials and metamaterials.

1.4.1 Polarization

Let us take a closer look at a few of Maxwell's equations. For a given material of volume V , we know that the total electric flux through its surface S is proportional to the total charge within that volume. This can be stated as

$$\int_S \mathbf{E} \cdot d\mathbf{s} = \frac{1}{\epsilon_0} \int_V \rho dv, \quad (1.8)$$

where \mathbf{s} is the normal to the surface S , and dv simply represents the incremental volume elements contained within the surface. We can use Gauss' law for electrostatics [Eq. (1.5a)], with a simple substitution, $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$, which leads to

$$\int_S \mathbf{E} \cdot d\mathbf{s} = \int_V \nabla \cdot \mathbf{E} dv. \quad (1.9)$$

We can show that \mathbf{E} is a conservative field, thus we can say that for an electric potential φ , $\mathbf{E} = -\nabla\varphi$. Once again, a simple substitution leads us to $-\nabla^2\varphi = -\rho/\epsilon_0$. For dielectrics, which have bound charges that make the predominant contributions to the material interaction with incident electromagnetic radiation, we can expand this out, leading us to a new term \mathbf{P} , or the polarization vector:

$$\epsilon_0 \nabla \cdot \mathbf{E} = \rho - \nabla \cdot \mathbf{P}. \quad (1.10)$$

A bit more insight may be found by rearranging as follows:

$$\epsilon_0 \nabla \cdot \mathbf{E} + \nabla \cdot \mathbf{P} = \rho. \quad (1.11)$$

We will assume for the moment that the polarization vector is linear, and we can make one last change to get a better idea how the polarization vector will affect the incident electromagnetic radiation:

$$\begin{aligned} \nabla \cdot [\epsilon_0 \mathbf{E} + \mathbf{P}] &= \rho \\ \rightarrow \epsilon_0 \mathbf{E} + \mathbf{P} &= \mathbf{D}, \end{aligned} \quad (1.12)$$

where \mathbf{D} is known as the flux density or electric displacement vector. It is clear now that, for a dielectric, the polarization vector is the only material property that affects the incident wave. We will take a closer look at the polarization vector and its nonlinear properties in Section 2.3.5.

1.4.2 Conductivity

Denoted here by σ_e , conductivity is defined as a measure of the freedom of electric charges to move in a given material. Conductivity specifically refers to ‘free electrons’ that travel across the bulk material versus the ‘bound charges’ that are associated with polarizability and are anchored locally by a restoring force. The terms free and bound are questionable here as all materials can, with high enough field strengths, be coerced into having conducting electrons. Both types of charges will be discussed in detail later in Section 2.2. We can see a consequence of the electric conductivity’s impact on an electromagnetic interaction through Ampere’s law [Eq. (1.5c)]:

$$\nabla \times \mathbf{H}(\mathbf{r}, t) = \frac{\partial}{\partial t} \mathbf{D}(\mathbf{r}, t) + \mathbf{J}_e(\mathbf{r}, t). \quad (1.13)$$

In this case, $\mathbf{J}_e(\mathbf{r}, t) = \sigma_e \mathbf{E}_e(\mathbf{r}, t)$.

1.4.3 Dispersion

Dispersion is the frequency-dependent description of how materials interact with electromagnetic radiation differently at different frequencies. We consider linear dispersion models for typical dielectrics and metals, as this approximates their behaviors quite well for most of the electromagnetic spectrum. Dispersion can be expressed by several quantities that can be compared versus frequency, to give a general appreciation for material interaction in some bandwidth or frequency range. (Also referred to as dispersion relations are the curves relating the magnitude of the wave vector $k = 2\pi/\lambda$, where λ is the wavelength, to the angular frequency $\omega = 2\pi f$, with f equal to the frequency of oscillation.) It is convenient to look at dispersion curves as a function of the real or imaginary parts of the index $\Re\{n\} = \lambda_0/\lambda$, where λ_0 is the free-space or vacuum wavelength, or we can look at dispersion curves as a function of the permittivity or permeability; at higher frequencies the permeability is typically near unity, and we have $\Re\{\sqrt{\epsilon}\} \approx \Re\{n\} = \lambda_0/\lambda$ see Figs. 1.1 and 1.2.

A more in-depth discussion of linear and nonlinear dispersion can be found in Section 2.2.1 or in Refs. 8 and 8.

1.4.4 Permittivity

The importance of permittivity to electromagnetic properties has been the primary focus of optics for well over a century, since at optical frequencies one typically assumes there to be little magnetic response. The term dielectric constant was widely used synonymously to relative permittivity.¹⁰ We refer to the former, the dielectric constant, as a zero-frequency parameter. Permittivity is generally assumed to be the dominant, if not the only, contribution to the

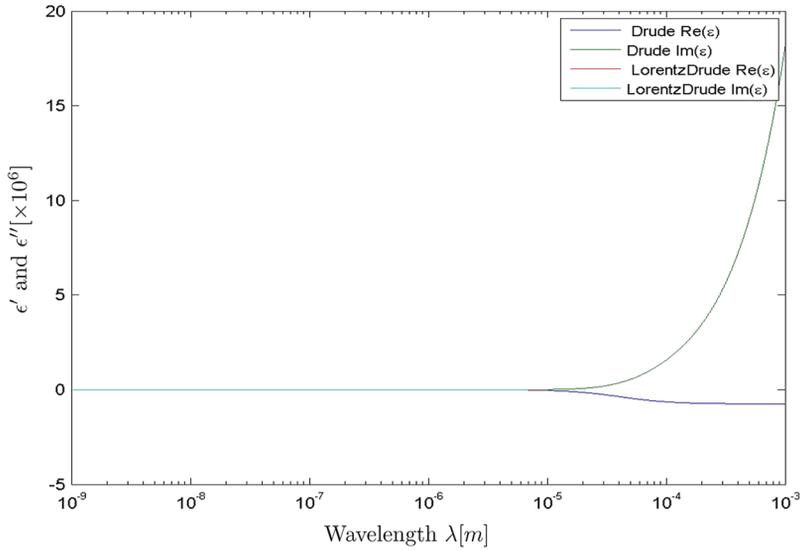


Figure 1.1 Broadband dispersion behavior of copper. In general, metals are described well by simple dispersion models, as shown by the relative close agreement between the Drude and the more complex Lorentz–Drude models here. Note that the large scale (10^6) can obscure the fact that both models are trending to unity at optical wavelengths.

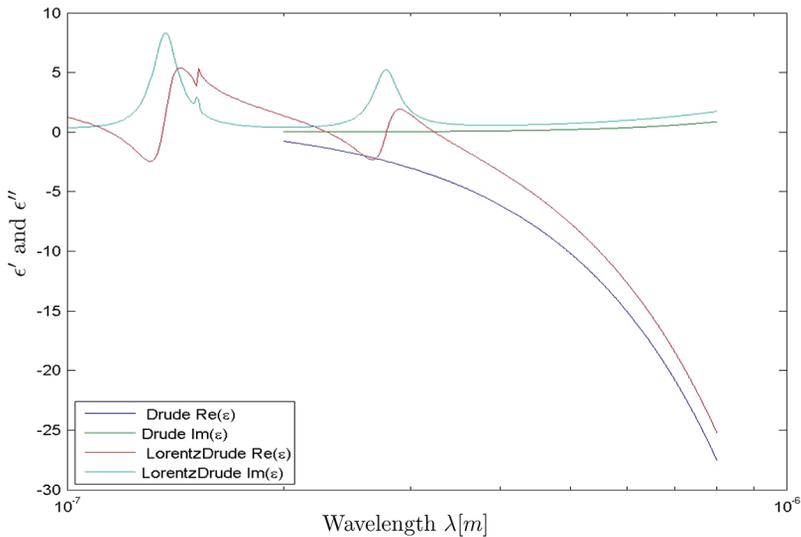


Figure 1.2 Near-resonance dispersion behavior of silver. A closer look illustrates the complex nature of near-resonance dispersion, where the Lorentz–Drude model more accurately depicts the real material. We will see that this complexity provides opportunities for engineering material properties.

index of refraction of a bulk material at optical frequencies, but part of the allure of metamaterials is the ability to engineer structures that can exhibit a magnetic response, even at optical frequencies. We will see later how this is achieved in metamaterials. Given that ϵ_0 is the permittivity of free space and χ_e is the electrical susceptibility, we can define the permittivity through this relationship while keeping in mind its frequency-dependent nature. We begin by using Ampere's law in differential form:

$$\nabla \times \mathbf{H}(\mathbf{r}, t) = \frac{\partial}{\partial t} \mathbf{D}(\mathbf{r}, t) + \mathbf{J}_e(\mathbf{r}, t). \quad (1.5c)$$

In order to make an association at a specific frequency or frequency ranges, we will consider a steady state case. The general form for a harmonic field with frequency given as ω is¹¹

$$\Re\{\mathbf{D}(\mathbf{r})e^{-i\omega t}\} = \mathbf{D}(\mathbf{r}, t). \quad (1.14)$$

This follows for $\mathbf{E}(\mathbf{r}, t)$, $\mathbf{H}(\mathbf{r}, t)$, $\mathbf{B}(\mathbf{r}, t)$, $\mathbf{J}_e(\mathbf{r}, t)$, $p(\mathbf{r}, t)$, ... as well. Using Eq. (1.5c) we can easily see that

$$\Re\left\{\left[\nabla \times \mathbf{H}(\mathbf{r}) - \frac{\partial}{\partial t} \mathbf{D}(\mathbf{r}) - \mathbf{J}_e(\mathbf{r})\right]e^{-i\omega t}\right\} = 0. \quad (1.15)$$

We can now substitute $\frac{\partial}{\partial t} \mathbf{D}(\mathbf{r})e^{-i\omega t} = -i\omega \mathbf{D}e^{-i\omega t}$ and

$$\nabla \times \mathbf{H}(\mathbf{r}) = -i\omega \mathbf{D}(\mathbf{r}) + \mathbf{J}_e(\mathbf{r}). \quad (1.16)$$

By definition, in a source-free region, the free current density $\mathbf{J}_e(\mathbf{r}) = \sigma_e \mathbf{E}(\mathbf{r})$, where σ_e is the electrical conductivity. We can also define \mathbf{D} as

$$\mathbf{D}(\omega) = \epsilon_0 \mathbf{E}(\omega) + \epsilon_0 \chi_e \mathbf{E}(\omega), \quad (1.17)$$

with ϵ_0 equal to the permittivity of free space, and where χ_e is what is called the electric susceptibility. We can now fully define the permittivity through this relationship:

$$\epsilon(\omega) = \epsilon_0[\epsilon_\infty + \chi_e(\omega)] + \frac{i\sigma_e}{\omega} \quad (1.18)$$

$$\epsilon(\omega) = \epsilon_0[\epsilon_\infty + \chi_e(\omega)] + \frac{i\sigma_e}{\omega} = \epsilon_0 \epsilon_r(\omega). \quad (1.19)$$

The complicated nature of $\epsilon(\omega)$ can be seen from Eq. (1.19). We note that the imaginary part of the electric susceptibility can incorporate contributions due to the bulk conductivity of the material, if it is present. This makes for some confusion as permittivity for metals, semiconductors, and dielectrics are often treated from separate theoretical standpoints. We look at some of these theoretical differences in Sections 2.2 and 2.3.

1.4.5 Birefringence

Consider the constitutive relationship between \mathbf{D} and \mathbf{E} shown in Eq. (1.6), where the permittivity is not isotropic. This relationship therefore can be represented by a tensor $\hat{\epsilon}$. We can write

$$\hat{\epsilon}\mathbf{E} = \mathbf{D}. \quad (1.20)$$

For a harmonic time-dependent wave equation, Eqs. (1.5a)–(1.5d) lead to

$$\begin{aligned} \mathbf{k} \times \mathbf{E} &= \omega\mu_u\mathbf{H} \\ \mathbf{k} \times \mathbf{H} &= -\omega\mathbf{E}. \end{aligned} \quad (1.21)$$

Then substituting

$$\mathbf{k} \times \mathbf{k} \times \mathbf{E} = -\omega^2\mu_u\mathbf{D}, \quad (1.22)$$

which, using Eq. (1.21) gives

$$\mathbf{k} \times \mathbf{k} \times \mathbf{E} = -\omega^2\mu_u\hat{\epsilon}\mathbf{E}. \quad (1.23)$$

The solution to this equation is three linear equations giving the components of \mathbf{E} and can be represented in matrix form, i.e.,

$$\begin{bmatrix} n_1^2k_0^2 - k_2^2 - k_3^2 & k_1k_2 & k_1k_3 \\ k_2k_1 & n_2^2k_0^2 - k_1^2 - k_3^2 & k_2k_3 \\ k_3k_1 & k_3k_2 & n_3^2k_0^2 - k_1^2 - k_2^2 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \quad (1.24)$$

In a uniaxial crystal, $n_1 = n_2 = n_0$ and $n_3 = n_e$, which simplifies the above matrix to

$$(k^2 - k_0^2n_0^2) \left(\frac{k_1^2 + k_2^2}{n_e^2} + \frac{k_3^2}{n_0^2} - k_0^2 \right). \quad (1.25)$$

The bracketed terms on the left represent free-space propagation, while on the right we have the dispersion relation required to determine (k_1, k_2, k_3) :

$$\frac{k_1^2 + k_2^2}{n_e^2} + \frac{k_3^2}{n_0^2} = k_0^2, \quad (1.26)$$

or for nonmagnetic media, where $\mu_r = 1$, and $n = (\epsilon)^{\frac{1}{2}}$, we have

$$\frac{k_1^2 + k_2^2}{\epsilon_{\perp}} + \frac{k_3^2}{\epsilon_{\parallel}} = k_0^2. \quad (1.27)$$

This describes an ellipse or equifrequency surface, i.e., the allowed k values in different spatial directions for a given frequency. In a biaxial crystal, $n_1 \neq n_2 \neq n_3$.

1.4.6 Permeability

As mentioned above, a material's permeability is often ignored in optics (i.e., at visible or 'high' frequencies). This is the case for materials having a permeability near unity and materials that are governed solely by their electrical response. Permeability plays an essential role in defining unique metamaterial properties not readily available in nature.

Using Faraday's law, with the addition of a magnetic current density term, we have

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = \frac{\partial}{\partial t} \mathbf{B}(\mathbf{r}, t) + \mathbf{J}_m(\mathbf{r}, t). \quad (1.5d)$$

As we have stated previously in Eq. (1.14), we would like to move to the frequency domain, which leads us to

$$\mathfrak{R}\{\mathbf{B}(\mathbf{r})e^{-i\omega t}\} = \mathbf{B}(\mathbf{r}, t) \quad (1.28)$$

for some frequency ω . We are again led to the equation for the real part, this time for Faraday's law:

$$\mathfrak{R}\left\{\left[\nabla \times \mathbf{E}(\mathbf{r}) - \frac{\partial}{\partial t} \mathbf{B}(\mathbf{r}) - \mathbf{J}_m(\mathbf{r})\right]e^{-i\omega t}\right\} = 0. \quad (1.29)$$

The simple substitution of $\frac{\partial}{\partial t} \mathbf{B}(\mathbf{r})e^{-i\omega t} = -i\omega \mathbf{B}e^{-i\omega t}$ leads to

$$\nabla \times \mathbf{E}(\mathbf{r}) = -i\omega \mathbf{B}(\mathbf{r}) + \mathbf{J}_m(\mathbf{r}). \quad (1.30)$$

By definition, in a source-free region, the free current $\mathbf{J}_m(\mathbf{r}) = \sigma_m \mathbf{H}(\mathbf{r})$, where σ_m is now the magnetic conductivity. We can also define \mathbf{B} as

$$\mathbf{B}(\omega) = \mu_0 \mathbf{H}(\omega) + \mu_0 \chi_m \mathbf{H}(\omega). \quad (1.31)$$

This completes the expression for the permeability as a function of μ_0 , which equals the permeability of free space, and χ_m , which is the magnetic susceptibility:

$$\mu(\omega) = \mu_0[\mu_\infty + \chi_m(\omega)] + \frac{i\sigma_m}{\omega}. \quad (1.32)$$

A material's magnetic response can be considered as originating from circulating currents, large or small, which in turn depend on the effective permittivity of the material, i.e., its bound and free charges, and their ability to move. The general expressions of the permittivity and permeability for a material or metamaterial are quite complex.

1.4.7 Index of refraction

More common is the concept of a material's index of refraction n . This quantity is deceptively simplified for most optical applications by being represented as a real positive number greater than or equal to 1. We will see that removing this simplification is a source for much of the surprising attributes of, and concurrently much of the opportunities associated with, the field of engineered materials, such as the properties illustrated in Table 1.1. Since n is dimensionless, we can only define it based on a relational equation, which is normally done through a comparison with the properties of electromagnetic waves in vacuum. Although the index of refraction was originally defined through an electric field interaction only, we must include the magnetic components of materials, as these will play a larger role than typically assumed in most materials:

$$n(\omega) = \sqrt{\mu_r(\omega)\epsilon_r(\omega)}. \quad (1.33)$$

We can plot material properties based on their values of the real parts of ϵ and μ (see Fig. 1.3). The double-negative region was predicted by Veselago to support propagating waves similar to the 'normal dielectric' quadrant.

1.4.8 A phenomenological description of refractive index

It is important to remember that only accelerating charges give rise to radiation. Imagine a charge at rest and its electric field lines as shown in

Table 1.1 Changes in some simple relationships required after the introduction of the magnetic properties of materials. We set $\zeta = \sqrt{\frac{\mu}{\epsilon}}$. We also note here that if $\epsilon < 0$ and $\mu < 0$, then $n < 0$, which is discussed in Chapter 6.

Physical laws	Equation with no magnetic response	Full equation
Snell	$\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$	$\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1} = \sqrt{\frac{\epsilon_2 \mu_2}{\epsilon_1 \mu_1}}$
Doppler effect	n	$\sqrt{\epsilon \mu}$
Cerenkov effect	n	$\sqrt{\epsilon \mu}$
Fresnel equations	$r_{\perp} = \frac{n_1 \cos \theta_1 - n_2 \cos \theta_2}{n_1 \cos \theta_1 + n_2 \cos \theta_2}$	$r_{\perp} = \frac{\zeta_2 \cos \theta_1 - \zeta_1 \cos \theta_2}{\zeta_2 \cos \theta_1 + \zeta_1 \cos \theta_2}$
Fresnel equation at normal incidence	$r = \frac{n_1 - n_2}{n_1 + n_2}$	$r = \frac{\zeta_2 - \zeta_1}{\zeta_2 + \zeta_1}$
No reflections	$n_1 = n_2$ \perp polarization	$\zeta_1 = \zeta_2$ $\sin \theta_{B\perp} = \sqrt{\frac{1 - (\mu_1 \epsilon_2 / \mu_2 \epsilon_1)}{1 - (\mu_1 / \mu_2)^2}}$
Brewster's angle	$\tan \theta_B = \frac{n_2}{n_1}$ \parallel polarization	$\sin \theta_{B\parallel} = \sqrt{\frac{1 - (\mu_2 \epsilon_1 / \mu_1 \epsilon_2)}{1 - (\epsilon_1 / \epsilon_2)^2}}$

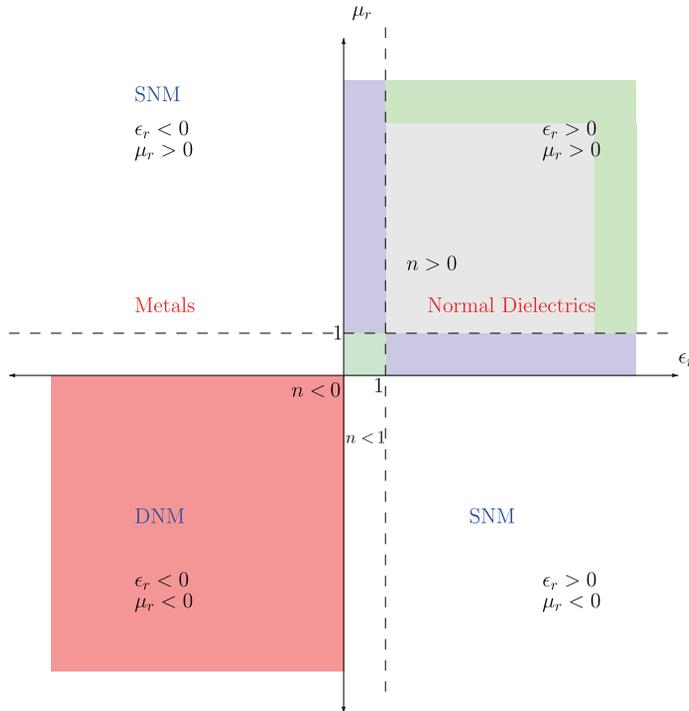


Figure 1.3 Permittivity and permeability domains. These domains explore the wide range of properties not normally associated with natural materials and the new and large number of possibilities that engineered materials offer, SNM is a single-negative-index material, and DNM denotes a double-negative-index material.

Fig. 1.4. If the charge is suddenly accelerated, the field lines near the charge are still radial, but at larger distances the information about the new position of the charge is not known because of the finite speed of light. In the diagram, we see that as the charge moves up, continuity of field lines dictate that there must be a downward-pointing transverse component to the field radiating away from the charge. This defines the source's radiation pattern, which will be at the same frequency as the external field.

In natural materials and metamaterials, charges are accelerated as a result of the external propagating field. The radiated waves contribute to a new field distribution governed by the nature of the induced currents, and these re-radiated waves will interact with neighboring units. The refractive index is, in some respects, a statement about the nature of the re-radiated light from a collection of units possessing currents induced by the electromagnetic wave. These might radiate as simple dipoles or more complex structures (e.g., quadrupoles, multipoles, etc.).

Since the radiated field comes from a large collection of oscillating charges (usually modeled only as dipoles) in a medium, why is refractive index n usually >1 in naturally occurring materials? Assuming a plane wave

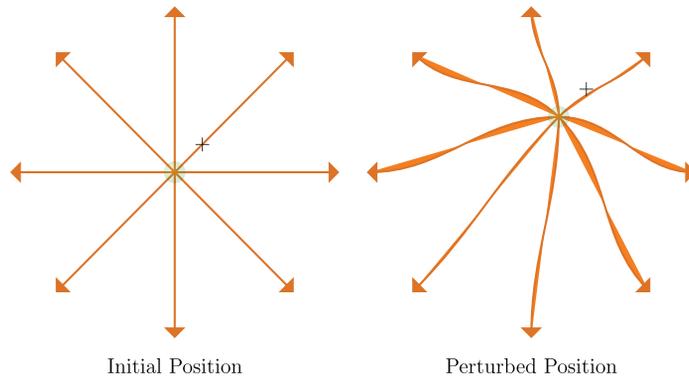


Figure 1.4 Re-radiation of a single charged particle. A stationary charge with its electric field lines is disturbed through an external field, which causes portions of its electric field lines to be disturbed, and even at points, turn perpendicular to the initial field lines. The relaxation time and subsequent re-radiation will control the material interaction with electromagnetic radiation.

incident on many subwavelength-sized units, like atoms or meta-atoms, the radiation drives a number of units into oscillation. The radiated field is typically in phase with the incident field in naturally occurring materials. (In a metamaterial it does not have to be!). If one calculates the field at a point from a sheet of dipoles, one finds that the lateral displacement between units usually leads to an effective 90-deg phase retardation with the incident wave. This is illustrated in Fig. 1.4. At some observation point, radiation from sources farther away arrive later, causing a phase delay. Integrating over a large sheet of dipoles leads to a total phase delay of exactly 90 deg compared to the incident wave. This corresponds to an index >1 , and the more units that contribute the larger the magnitude of that index. Effective phase retardations or advances depend on the sources of the radiated field.

If the charge motion has a resonance, then the phase shift of the radiated field with respect to the incident field can vary from 0 to 180 deg. As we will see in detail, these phase shifts occur through an absorption (or gain) resonance as a function of frequency. For real materials, depending on the frequency of light with respect to nearby resonances, we see an advance in the phase or a delay. This is illustrated in Figs. 1.5 and 1.6. We interpret these phase shifts either in terms of a slow down or speed up of the light (i.e., an increasing or decreasing phase velocity).

In a meta-atom one can engineer very large resonances and ensure a high degree of coherence between the resonant units, leading to much larger effective index values compared to most conventional materials. One can also use such effects to take the effective index back down through zero to negative values.

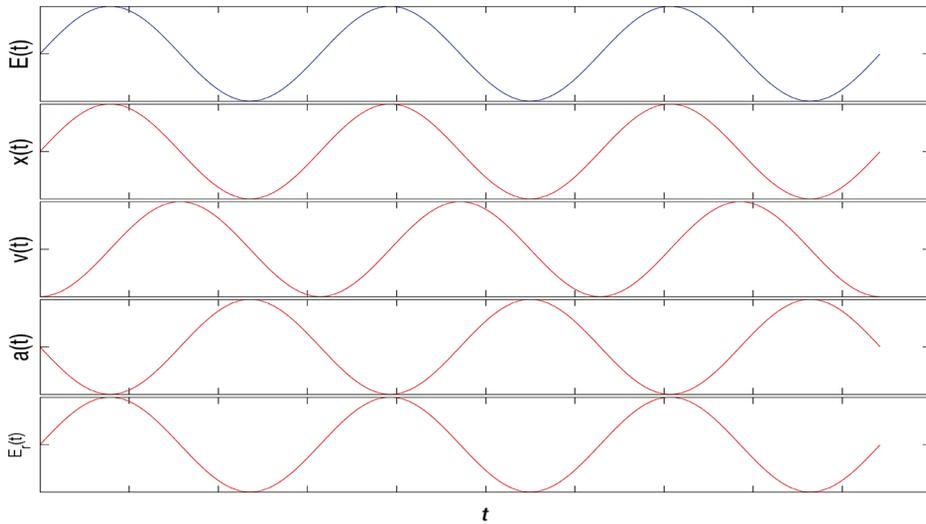


Figure 1.5 Position, velocity, and electric field field for $n = 1$. Continuity of field lines dictates that there must be a downward-pointing transverse component to the field radiating away from the charge. This defines the source's radiation pattern, which will be at the same frequency as the external field. For free space, $n = 1$, we would envision the associated charge's position $\mathbf{x}(t)$ as being perfectly in phase with the incoming radiation $\mathbf{E}(t)$, thus our re-radiated wave $\mathbf{E}_r(t)$ is also perfectly in phase.

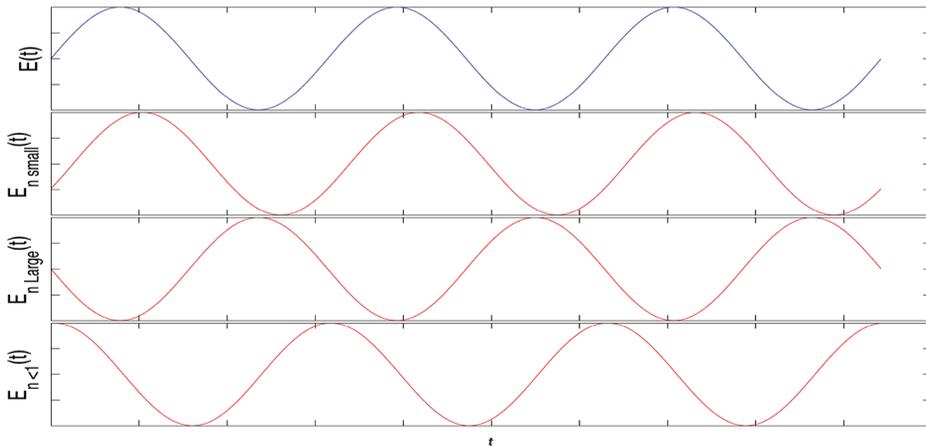


Figure 1.6 Electric field phase based on index. The refractive index is a statement about the nature of the re-radiated light from a collection of units possessing induced currents. A phase delay for $\mathbf{E}_{n\text{small}}$ and $\mathbf{E}_{n\text{large}}$, or advance in the case for $\mathbf{E}_{n<1}$, is expected when compared to the incident field. These might radiate as simple dipoles or more complex structures.

1.5 The Six Velocities of Light

Long before Veselago's paper was 'rediscovered,' there were theoretical descriptions of light traveling within a medium that worked out the physics of the propagation of electromagnetic waves within and through some medium.

Many of these descriptions are discussed in detail in a text written by Brillouin (with acknowledgments to the chapter by Sommerfeld)¹² as a theoretical construct for the description of electromagnetic waves, and in doing so illuminate the reasons a negative index does not violate causality or other physical laws. Metamaterials are necessarily dispersive, which dictates both the group velocity v_g and the phase velocity v_p . A closer look into these propagation phenomena in dispersive media is fundamental to ensuring that no nonphysical results appear from theoretical models.

1.5.1 Free-space velocities

Following the notation used by Brillouin, we know that

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}, \quad (1.34)$$

where ϵ_0 and μ_0 are the permittivity and permeability of free space, respectively, as defined immediately following Eqs. (1.5a)–(1.5d). Although it may seem possible initially through metamaterial designs, it is important to note that based on special relativity, information in a wave cannot travel faster through space than this value.

The speed at which the phase of a specific frequency ω_0 travels through space is defined as the phase velocity:

$$W = v_p = \frac{\omega}{k}, \quad (1.35)$$

where ω is an individual frequency with $k = \frac{2\pi}{\lambda}$, with k being the wavenumber, and λ is the wavelength of the individual frequency. In free space Eq. (1.35) is simply $v_p = c$. This velocity can be thought of as the motion of elementary wavelets in the carrier depicted in Fig. 1.7.

The group velocity v_g is defined as the speed at which a larger disturbance, created by a packet or collection of waves with finite bandwidth, travels through space. In free space the group velocity is not dependent on the collection of frequencies that make up the wave packet, since

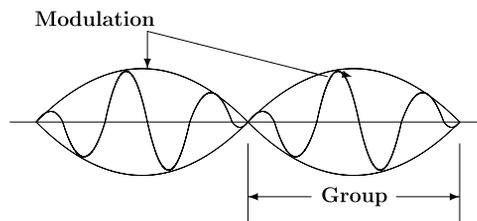


Figure 1.7 Wave packet showing a combination of waves resulting in an envelope modulation. These individual waves, each moving at their own v_p , of a carrier are causing the larger modulation, which move at v_g . (Adapted from Ref. 12.)

$$U = v_g = \frac{\partial \omega}{\partial k}. \quad (1.36)$$

The group can be thought of as the modulation impressed on the carrier and is the result of a building up of some of the groups of individual wavelets into a large amplitude that moves along with velocity v_g .

We can now define the signal velocity, which governs the transfer of information through a medium. Signal velocity S can be defined as the moment when forced oscillations of the characteristic frequency of the incoming wave become detectable in a medium. Generally speaking, the signal velocity refers to the moment when oscillations are on the order of the magnitude of the input signal, but this does not always need to be the case, as with a lossy material. In free space, once again,

$$S = c = v_p. \quad (1.37)$$

Energy velocity can also be defined depending on the medium. In a nondispersive medium, or a medium with absorption far from the frequency of the incoming signal, energy velocity can be defined as the group velocity. However, in a dispersive medium, the energy velocity has to be defined as the ratio of the Poynting vector to the energy density.

We can now define the last primary velocity, which is the front velocity. The front velocity, or the velocity of the wavefront, is defined as the speed at which the smallest and most minute disturbance of the field propagates through a medium before the medium has time to respond. Since the energies at the wavefront are so small, they are effectively undetectable, thus representing a rather theoretical space where the wave exists. In Brillouin's¹² derivation, the wave requires a finite beginning and end; otherwise, most of the previous definitions become meaningless since the wave would otherwise exist everywhere throughout space. In this case, the front velocity also has a counterpart called the end velocity, which is created by the beginning of a new wave of opposite phase but equal shape and amplitude. Through destructive interference, these amplitudes cancel, thus equivalently destroying all oscillations in the carrier. Both of these are needed, as they set limits on the other velocities.

1.5.2 Waves in a medium

The value of c was defined in vacuum and does not change in a medium. It is v_p that is a function of ω or k and that is dependent on the type of medium. Lord Rayleigh used ocean waves as an example, showing that the group velocity and phase velocity did not have to equal c . For electromagnetic waves, phase velocity is determined by the reaction of local oscillators. In typical metals we consider this to be made up of electrons, which usually have a characteristic plasma frequency in the UV range. In the case where the phase

velocity is near a local oscillator's characteristic frequency or a resonance, the phase velocity is now a function of the local oscillators in the medium; thus, $v_p \rightarrow v_p(k, \rho, T)$ with $k = 2\pi/\lambda$ as the wavenumber, ρ as the density of oscillators, and T as the temperature. We can now also look at the group velocity in a medium:

$$v_g = \frac{\partial \omega}{\partial k} = \frac{\partial k v_p}{\partial k} = v_p + k \frac{\partial v_p}{\partial k}. \quad (1.38)$$

Demonstration of differences in signal, group, and phase velocities can be shown through the different paths of integration of two waves of opposite phase but equal amplitude, one beginning at time $t = 0$, the other at time $t = T = n\tau$ with τ equal to a single period (which is shown graphically in Fig. 1.8):

$$f(t) = \frac{1}{\tau} \int_{-\infty}^{\infty} \frac{dn}{n^2 - \left(\frac{2\pi}{\tau}\right)^2} [e^{in(t-T)} - e^{int}] \quad (1.39)$$

The most complicated velocity to predict in a dispersive medium is the signal velocity. This requires rigorous work to define the time and space between the wavefront and the onset of the first forced oscillations after other sets of motions called forerunners. The forerunners exist before the forced oscillations, thus they do not have the characteristic frequency of the signal and in a dispersive medium actually follow two different paths. An example of these forerunners is shown in Fig. 1.9 and further discussed in Ref. 12.

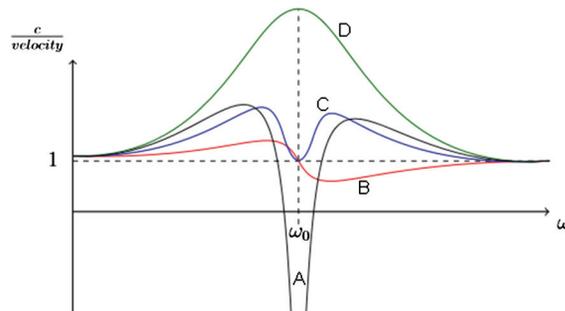


Figure 1.8 Graphical representation of several velocities of light. This representation is a result of taking different paths of integration from Eq. (1.39). A represents c/W , where $W =$ phase velocity; B represents c/U , where $U =$ group velocity; C represents c/U , where $S =$ signal velocity; and D represents c/U_1 , where $U_1 =$ energy velocity. (Adapted from Ref. 12.)

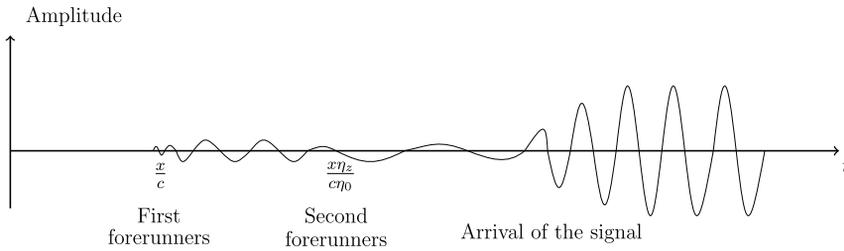


Figure 1.9 Schematic of wave propagation in a medium. The signal velocity can be differentiated by the first two groups of forerunners through its characteristic frequency and magnitude. (Adapted from Ref. 12.)

1.5.3 Superluminal speeds

The initial interest in metamaterials was to create phase velocities or group velocities not found in nature that would clearly include those faster than light or superluminal. There is no restriction on v_p or v_g in a dispersive medium, but this still does not imply that the signal velocity is greater than c , as Fig. 1.10 illustrates. This important consequence also shows some of the theoretical limits of our previous derivations since we have, for the most part, neglected the time evolution of the wave and considered only steady state solutions.

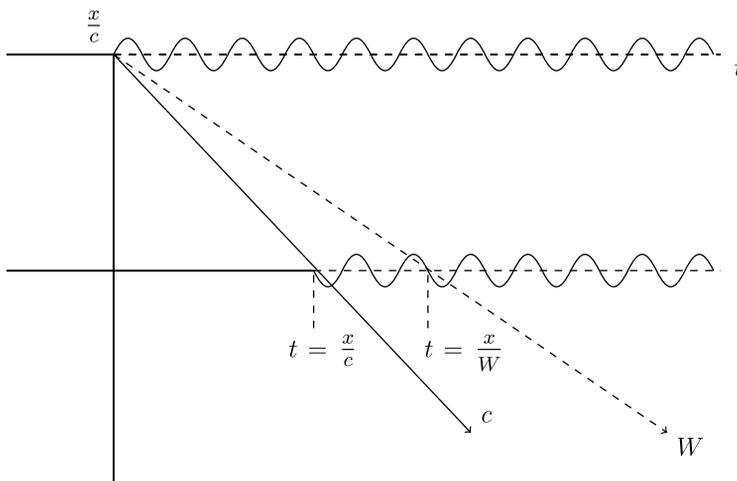


Figure 1.10 Illustration of propagation in a medium. This is a demonstration of the propagation speeds of the phase and wavefront, or c as a function of distance in a medium. (Adapted from Ref. 12.)

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