Visual Optics

LECTURES IN OPTICS
Volume 4

George Asimellis
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**FOREWORD**

The application of core vision science to clinical practice is universal, yet finding a suitable teaching resource to convey a comprehensive message is difficult. Students may be forced to rely on superficial knowledge gained through Internet sources due to a lack of suitable resources. What is required is a textbook that has a comprehensive discussion on a range of core vision science topics that provides a nexus to understand basic principles essential for clinical practice. The *Visual Optics* text of Dr. Asimellis is such a text, providing sections devoted to the development of the eye, monocular sensory perception that includes visual acuity, contrast sensitivity, color vision, and then an in-depth discussion on refractive errors and optics of the eye and core optics concepts.

The textbook beautifully integrates biological science, physical science, and clinical science to produce a textbook that would be useful to the novice and to those with a keen interest in optics and the eye. Dr. Asimellis’ expertise in visual optics is clearly illustrated in the textbook, which challenges those who may be outside their knowledge comfort zone.

The text begins with basic optics and vision principles, including comparative anatomy of the eye. There are introductory examples of image formation and the visual pathways, which are useful in providing a holistic view of vision. Basic optical principles are then integrated with visual perception to integrate this core knowledge. In addition to visual acuity, the perception of color is introduced, including connections to photoreceptor density and type, and how the encoding of chromatic information is achieved. The remaining text is focused on the optics of the eye, including a comprehensive discussion on ametropia and factors affecting image quality. The expertise of the author is clearly evident in this section, which includes excellent biometric measurements that are now essential in clinical practice.

Dr. George Asimellis has written a text from a student’s perspective and provides a well-laid-out and well-illustrated format that is comprehensive and highly readable. The integration of theoretical and clinical information contributes to the understanding of monocular sensory and clinical procedures. I congratulate him and have no reservations in recommending this textbook to all of those with an interest in vision science.

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Preface

Visual optics is, in essence, an application of optics for a very important sensory organ: the eye. While rather complicated, the process of vision can be divided into two parts, the optical and the neural. This book extensively and comprehensively covers the optical part, while providing a simple description of the neural part, which is restricted to those aspects that are critical to understanding visual function.

The topic of visual optics can be quite challenging and fascinating. This topic bridges knowledge acquired from more science-oriented, geometrical and wave optics material with the application of this knowledge to the eye, and extends in clinical relevance to optometry and ophthalmology. Optics is the foundation of how the eye works, how we image the eye for diagnosis, and, most recently, how we use many laser-based therapeutic and cosmetic applications. A good understanding of the simple yet powerful relationships that describe the interaction between light and the eye helps to pave the way to a comfortable approach to understanding the operation of traditional examination techniques such as retinoscopy as well as modern ocular diagnostics such as topography and optical coherence tomography. Thus, the first three books in this series provide the scientific foundation for the explorations and applications discussed in this book. The applications of ocular imaging are extensively presented in the final volume of this series, Ocular Imaging.

Combining optical science with clinical relevance, this Visual Optics volume is written specifically for adult learners in the optometry and ophthalmology professions. It is referenced with the most recent research findings that have been published in peer-reviewed journals—almost 1000 external references are included.

The text follows the didactic principles adopted throughout the series, with adherence to a deductive approach, lots of practical examples, ample illustrations, and clear, concise language. Often my students tell me that the words from the lectures are also the words in my texts. While I see this as an exaggeration, it is true that, despite endeavoring to explain complex concepts and the need to adhere to strict and rigorous definitions, simple language is always sought, with the goal of being thoroughly understood in a clinically meaningful way.

The book is organized based on two broad concepts. The first concept, spanning Chapters 1 to 5, pertains to the well-functioning eye, or the eye that produces retinal images of sufficient quality; let’s call this emmetropic visual optics. The second concept spans Chapters 6 to 10 and is called ametropic visual optics. This covers the science of spherocylindrical ametropias, i.e., myopia, hyperopia (Chapter 6), and astigmatism (Chapter 8) for distance- and near-vision accommodation (Chapter 7) and extends to aspects of the aging eye, which include presbyopia and low vision (Chapter 7), optical correction and its considerations (Chapter 9 on ophthalmic lens optics), and prismatic effects (Chapter 10).

Every chapter is followed by an extensive, multiple-choice quiz and a short summary. The quiz questions are in the format followed by the National Board of Examiners in Optometry (NBEO) and aim to be an element of self-evaluation and assessment for the reader.
The contents of this book can be used in a multitude of instructional courses, including core visual optics courses, as well as courses on perception and ophthalmic optics. A recommended structure that adheres best to the flow of this book could be as follows:

**Emmetropic Visual Optics;** 2-credit course / 30 lecture hours

- Unit 1, 3 hours: Optics of the Eye – Chapter 1 (including some Geometrical Optics review)
- Unit 2: 10 hours: Refractive Elements of the Eye – Chapter 2.
- Unit 3: 5 hours: Visual Acuity – Chapter 3 (including some Wave Optics review) and § 4.5 Digital Signal and Analysis
- Unit 4: 6 hours: Retina / Optics of the Retina – Chapter 4
- Unit 5: 6 hours: Color Science and Color Vision – Chapter 5

**Uncorrected & Corrected Ametropic Visual Optics;** 4-credit course / 50 lecture hours

- Unit 1, 8 hours: Depth of field, emmetropia, ametropia, myopia, hyperopia – Chapter 6
- Unit 2: 10 hours: Accommodation, near vision, presbyopia, low vision – Chapter 7
- Unit 3: 10 hours: Astigmatism: optical effects, visual effects, nomenclature, effects on vision – Chapter 8
- Unit 4: 14 hours: Ophthalmic Optics: vergence, lens powers, cardinal points, corrected retinal image, lens effects on vision, lens parameters, toric lenses, light transmission properties, neutralization – Chapter 9
- Unit 5: 8 hours: Prismatic effects: prism geometry and optics, prisms in vision correction, combinations of prisms, lens tilt and shift effects – Chapter 10

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October, 2021
1.1 THE FOUNDATIONS OF VISION

1.1.1 Our Understanding – The First Steps

Any attempt to explain the intricacies of vision in layman's terms is challenging because vision is a highly individualized and extremely complex process. Vision is a process that utilizes the two eyes, engages over forty different parts of the brain, and involves several hundred connection pathways within the brain.

The optics of the eye and the physiology of light transduction within the retina are perhaps essential aspects of vision. However necessary it may be to understand these aspects, it is important to acknowledge that vision is much more than just optics. Throughout this book, we will discuss optics, and although this book is long, it is not the full story.

The German astronomer Johannes Kepler was among the first to describe vision in a way that is close to our current understanding. In his book entitled *Ad Vitellionem Paralipomena, Quibus Astronomiae Pars Optica Traditur* (1604), Kepler describes the dioptrics of the eye using the image formation inside the eye. Just some years later, French philosopher,
accumulated information from multiple fixations, a stable **mental model** with high resolution throughout the field.

**Saccades** are the movements that our eyes make as they dart from one position to the next. They can be characterized as jerk-style, step-like, push–pull conjugate eye movements. Saccades help one to perceive a clear image across the entire field by instantly bringing a different part of the field to the fovea.⁹ On the other hand, **pursuits** are the smooth eye movements that we make when observing a moving target, in order to keep that moving target fixated on the fovea. Saccades are freely generated, but pursuits require a moving target in our visual field.

![Figure 1-6: Central vision versus peripheral vision.](image)

### 1.3 Image Perception and Encoding

From the initial stimulus of light, the eye must acquire the light photon, register/encode it, and preliminarily process it before sending it to the brain. The main refractive elements, the cornea and the crystalline lens, form a real image of the observed object on the retinal surface, and in particular, on the macular region. This is the first step of vision, the **formation** of an image.

The second step is the **perception** of the light by the light receptors in the retina, a thin layer of neural tissue, located at the back of the eye. This is the simplest, most elementary **form of vision**. Because it is formed on the retina, the image is called a **retinal image**. Its perception is due to photoreceptor responses that are communicated through several layers of retinal neurons.

The third step involves the **encoding** of the retinal image into a neural response by the photoreceptors, akin to an electrical signal that is sent from the retinal neurons to the cortical neurons. Thus, the photoreceptors are not just detection elements; they are also encoding and initial processing elements. In the human eye, these almost 100 million photoreceptors are

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the collected information is then interpreted by the brain as an image. Some elements of light guidance, such single or multiple lenses, were eventually developed.

Figure 1-24: (left) The compound eye in which light from a specific angle ‘activates’ a microscopic eye. (right) Anatomy of an ommatidium.

The compound eye has some advantages, such as directionality and increased field of view. The large compound eyes are immobile; however, because of their spherical protruding shape, they provide an almost 360° view. Thus, instead of the eyes moving, the compound eyes in insects receive information simultaneously from a very large view.

Several aspects of insect vision are fascinating. Pollinating insects, such as bees, possess color vision. These eyes, however, do not have the best resolving power in the animal kingdom. One arcminute of a human eye can make out fine details at just 1 meter away, whereas a corresponding portion of an insect eye at its best resolution, looking from the same distance, can only perceive an outline of the same shape.

In its simplest form, an ommatidium has some form of light-condensing optics, a lens, for example, and guidance, an optical shaft (rhabdom), which leads the gathered light to pigment cells that serve as sensors. Each ommatidium has its own nerve fiber connecting to the optic nerve, which relays information to the brain.

The evolutionary process of the eye in various species is particularly interesting. There is a multilayer aspect of eye evolution that involves some or all of these factors: (a) development of efficient photopigments at the photoreceptors, (b) improvement of directionality, and (c) development of the light-gathering and focusing optics. The photoreceptors, in particular, appear to have a common origin among the species. Most eyes that we recognize today can have their origins traced to in the Cambrian explosion, which took place about 550 million years ago, during which a rapid evolutionary progression occurred.

The main refractive elements of the eye are the cornea and the crystalline lens; the iris serves as the aperture stop. The transmission of light through the eye is influenced by the transparency of the ocular media, which also include the aqueous and the vitreous.

### 2.1 CORNEA

#### 2.1.1 Corneal Shape

The cornea is the exterior window of the eye and is almost completely transparent to visible light. It is the main refractive element in the optical system: The cornea is responsible for about \( \frac{2}{3} \) of the optical power of the eye (\( \approx +40 \, \text{D out of total } \approx +60 \, \text{D} \)). Because of this, the cornea is also the main source of refractive errors and high-order aberrations.

The cornea has a dome-like shape, maintained by the intraocular pressure (IOP). The cornea joins the sclera, the ‘white’ of the eye, at the corneoscleral limbus. The exterior profile of the cornea appears slightly elliptical, with an average diameter of 11.7 mm horizontally and 10.6 mm vertically. This difference is due to, among other factors, the upper eyelid pressure. The interior profile has a circular shape, with an average diameter also of 11.7 mm.\(^{38}\)

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**VISUAL OPTICS**

*Figure 2-12: Anterior (left) and posterior (right) corneal refractive power. The sum of the two is the corneal equivalent refractive power (the color-coded scale shown to the right of each image reads their values).*

Food for thought: What would the corneal power be if the cornea were surrounded by air?

The optical power of the outer, anterior corneal surface is unaffected. We use Eq. (2.2): \( F_{\text{anterior}} = (1.376 - 1.00)/(+0.0077 \text{ m}) = +48.83 \text{ D} \).

For the inner, posterior surface, we use a version of Eq. (2.3): \( F_{\text{posterior}} = (1.0 - 1.376)/(+0.0068 \text{ m}) = -55.29 \text{ D} \).

The two values add to a total of \(-6.46 \text{ D} \approx -6.50 \text{ D} \). In other words, if the cornea were completely surrounded by air, it would, indeed, be a negative lens of about \(-6.50 \text{ D} \) power.

*Figure 2-13: The corneal optical power is approximately the sum of the anterior and posterior optical powers. Because the medium to the right of the posterior surface has a refractive index that is almost the same as that of the cornea, the posterior surface optical power is \( \approx -6.0 \text{ D} \).*

We conclude therefore that the main contributor to the corneal refractive power is its anterior surface. This is the reason that the anterior corneal surface properties have such significance in visual and clinical optics.\(^{63}\)

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### 2.4.4 Angles in the Human Eye

The angle between the pupillary axis and the line of sight is **angle lambda** $\lambda$. Because the line of sight has a nasal inclination with respect to the pupillary axis, angle $\lambda$ tends to be displaced nasally, with values\(^{188}\) measuring from $3^\circ$ to $8^\circ$. Positive values indicate a nasal displacement, and negative values indicate a temporal displacement.

Angle lambda $\lambda$ is the only angle corresponding to eye anatomical landmarks because both of its sides are defined anatomically, lending itself thus to practical clinical measurements: The pupillary axis and the line of sight both pass through the center of the entrance pupil, which is the apex of angle $\lambda$. This angle can be observed and measured clinically as follows: The subject’s eye fixates on a point light source and the observer traces the corneal reflex of that source by moving the source laterally (typically, nasally) until the reflex is seen overlaid on the subject’s pupil center. The line from the observer and the subject’s pupil center is the pupillary axis, while the line from the light source and the subject’s pupil center is the line of sight.

**Angle alpha** $\alpha$ is the angle formed between the optical axis and the visual axis and is an optical approximation to angle $\lambda$.

![Figure 2-55: (left) Angles kappa $\kappa$ and lambda $\lambda$. (right) Angles alpha $\alpha$, lambda $\lambda$, kappa $\kappa$, and gamma $\gamma$ (typically) tend to be nasally displaced because of the temporal shift of the fovea.](image-url)

**Angle kappa** $\kappa$ is formed by the pupillary axis and the visual axis. Its apex is the object-space nodal point $N$. Unlike angle $\lambda$, angle $\kappa$ is fixed; i.e., it is not dependent on pupil centration variations. However, it is not possible to clinically measure angle $\kappa$ because the location of the nodal point is not defined by an eye landmark. Often, the numerical differences between angles $\kappa$ and $\lambda$ are very small, so both are sometimes referred to as angle $\kappa$.

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Thus, a black-and-white pattern will appear as gray-on-gray. Therefore, the chief visual effect of scatter is a reduction of contrast, which decreases vision at dusk and nighttime, increases the difficulty in night driving, and introduces some diurnal fluctuation in vision.

![Image](image.png)

**Figure 2-72:** Dr. Thomas JTP van den Berg shown with minimal (left) and significant (right) disability glare. Note the reduction in contrast. Images courtesy of Dr. Thomas JTP van den Berg, used with permission.

Scatter distribution and magnitude are different in each eye, and even differ between the eyes of the same individual.\(^\text{230}\) This difference depends on age, iris color, pathologies such as cataract or diabetes, and possible previous ocular surgery. Although low in a young, healthy eye, scattering is mainly attributed to centers in the cornea and the lens, while smaller amounts are attributed to the iris\(^\text{231}\) and the fundus. An unstable tear film also contributes to transient fluctuations of scatter.

**Figure 2-73:** Forward scatter mainly affects the contrast in the retinal image.

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**backward scatter**
- reduces the light forming the retinal image
- observed in a slit lamp
- quantified by Scheimpflug imaging

**forward scatter**
- affects vision
- disability glare
- discomforting glare

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Visual acuity is, in simple words, a measure of the clarity of vision, or how well a person sees. According to international standards, visual acuity is a numerical variable that characterizes the ability of the visual system to recognize optotypes.\footnote{246 http://standards.globalspec.com/std/10052053/din-en-iso-8596 and www.iso.org/obp/ui/#iso:std:iso:8596:ed-3:v1:en}

Often, erroneously, the terms ‘visual acuity’ and ‘vision’ are used interchangeably. Visual acuity is one of the standard parameters by which vision is evaluated. It directly relates to the ability of the optical system of the eye to resolve detail, depending on the clarity of the focused image. Visual sensation involves an intricate set of steps and relies on factors such as visual innervation and the cerebral interpretive faculty. Vision, in essence, is the ability of the brain to recognize the objects that form the retinal image.

Visual acuity relates to optical resolution, which can be expressed by the closest angular spacing of the two finest/closest targets that can be seen as two. The measurement of visual acuity often involves charts; hence, it relates the eye’s ability to either resolve targets, called optotypes, on a chart placed a large distance away from the subject, or to read text on a chart placed very close to the subject. Reading, however, involves, interpretation and recognition, in addition to resolution. Near vision and low vision, which are broader in perspective, are presented in the chapter on accommodation and presbyopia (§ 7.9 and § 7.10).
distinguish or even guess at. Küchler later introduced (in 1843) Blackletter/Old English letters (which formed words) arranged in twelve lines of diminishing size toward the bottom.

![Figure 3-20: Küchler's eye charts using words.](image)

In 1854, Austrian ophthalmologist Eduard Jäger (Jaeger) von Jaxthhal published a series of reading test cards called Schrift-Scalen that were originally an addendum to a book on cataract surgery to be used for near-vision testing (§ 7.10). The design of these cards was focused on determining an individual’s ability to function following cataract surgery, as part of an activity of daily life (ADL) assessment. Reading complete words is related to, but is not exactly, visual acuity testing (see low-vision testing, § 7.9) since words might be guessed from the passage.

Jaeger’s eye charts were the first successful charts of all kinds. They were published in many languages and had excellent print quality, as he used typefaces from the Vienna State Printing House, with well-specified linear print sizes. Jaeger never developed a distance test chart, as he did not prescribe a fixed reading distance.

The founding fathers of what we today call clinical measurement of (distance) visual acuity are two Dutch ophthalmologists, Fransicus (Franz) Cornelis Donders, who is credited with coining the term visual acuity, and Herman Snellen, who is credited with the first successful standardization of visual acuity measurement.266

Donders was interested in distance vision tests as a means to determine refractive error; his focus was on how the eyes function, as he was a pioneer of determining ocular refraction on a scientific basis. Initially, he used the larger Jaeger samples, but he needed a more scientific technique. In 1861, at the annual ophthalmology meeting in Heidelberg, Germany, Donders proposed a formula and a reference standard, defining, for the first time, sharpness of vision in terms of the ratio of letter size to specific viewing distance—an angular measure. His proposal was well-received.

The examination distance is assumed to be 20 ft; therefore, the numerator is always 20. At this distance, a person with standard visual acuity can read the smallest letter forming a total angle of 5 arcmin. This is the 20/20 visual acuity standard.

Example ➜: What is the visual acuity of a person who discerns an optotype 17.46 mm tall from 20 ft?

The height of the Snellen 20/20 optotype subtending an angle of 5 arcmin at 20 ft is 8.73 mm. The 17.46 mm optotype is 2× larger than the 20/20 optotype. This can be read by the standard eye at a distance of 40 ft, which means that the person’s Snellen visual acuity is 20/40.

Example ➜: Donald has a less-than-standard (reduced) vision. The smallest optotype he can read is a large one, which can be read by a standard vision eye at, say, 100 ft. Donald, then, has 20/100 vision, which is poor. He will probably fail to get a driver’s license in most of the United States, since the minimum required visual acuity for driving is 20/40.
3.4.4 Advanced Optotypes and Eye Charts

The test of visual acuity using familiar letters or numbers is based on alphanumeric characters. For this reason, it may be influenced by human subjective perception. The effort to disassociate this subjective aspect and further improve and standardize visual acuity testing has led to the development of various optotypes and eye charts, some of which are as follows:

The **Landolt C**, also known as a broken ring, is an optotype developed by the Swiss-born ophthalmologist Edmund Landolt in 1888. The C is a nearly complete ring with a gap and is thus identical to the sans-serif Snellen C (Figure 3-26). The gap is rotated by steps of 90° and in some variations by 45°. The Landolt C is the standard optotype, together with measurement procedures as described in International Organization for Standardization ISO 8596, which pertains to ophthalmic optics, visual acuity testing, and standard and clinical optotypes and their presentation.\(^{246}\)

The **tumbling E** 5×5 grid consists of rows of a letter-E-like sans-serif optotype rotated by increments of 90°. Rather than being a letter E, it is more like a vertical line with three horizontal lines (bars) attached; the three bars approximate part of a square-wave grating.\(^{292}\) The tumbling E chart is suitable for populations not native to the Latin (or any similar) alphabet as well as those who are illiterate very young. When the chart comprises either the Landolt C or the tumbling E, the examinee is asked to show the orientation of the C gap or the E lines.

![Figure 3-41: The Landolt C and tumbling E eye optotypes. When used at the 20/20 line, the angular extent of the diameter of the C character and the height of the E character are each 5 arcmin.](image)

The **LEA Vision Test System** is a series of vision charts and tests designed specifically for children, using pictorial optotypes—outlines of an apple, a circle, a house, and a square. The LEA test was introduced in 1976 by Finnish pediatric ophthalmologist Lea Hyvärinen.\(^{293}\) It allows pediatric low vision to be diagnosed at much younger ages than standard vision tests allow.

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Many similarities and differences exist in the way an artificial optical system, such as a photo camera, and the human eye record and perceive (or process) images. In a camera, this role is undertaken by the film or the digital sensor. In the eye, this role is carried out by the retina.

### 4.1 Retinal Structure, Geometry, and Optics

The retina is the photosensitive part of the eye. It is a complex, semitransparent multilayer tissue located at the back of the eye (fundus), covering the choroid. The retina surface spans about 65% of the eye’s interior and is centered about the posterior pole (central retina). The outer periphery of the retina reaches about 9 mm temporally and 15 mm nasally along the horizontal meridian. The junction between the retina and the ciliary body is known as the *ora serrata*. The ciliary body covers the remaining 35% of the eye’s inner lining surface area.

The main landmarks in the human retina are the *area centralis*, the *optic disk*, and the retinal blood vessels. The area centralis is the most sensitive part of the retina, consisting of the *macula*, its central part the *fovea*, and its most central part, the *foveola*. The retinal blood vessels, particularly the central retinal vein and the central retinal artery, converge to the optic disk and are visible fundus landmarks.
Fundus photography has been the primary method of documenting retinal structure. Its key advantage is that the retina can be photographed directly since the pupil is used as both the entrance (illuminating) and the exit (imaging) paths. Fundus photography allows for evaluation of the fine details of the anatomy and monitoring of the longitudinal health status of the retina.

Figure 4-3: (left) Color fundus photography of a left eye extending perimetrically to 30°. (right) Fundus photo using specialized dyes, including fluorescein and indocyanine green, which emphasize the retinal vessel structure (fluorescein angiography).

Computerized imaging techniques, such as optical coherence tomography (OCT) and confocal scanning laser ophthalmoscopy (CSLO) further enhance retinal imaging possibilities. OCT imaging produces quantitative data about macular or retinal thickness and qualitative features relating to retinal and subretinal morphology. Owing to the high-speed imaging capability of Fourier-domain OCT, current devices can reconstruct an in vivo 3-D representation of the subretinal morphology with an axial resolution of a few microns.

Figure 4-4: (left) Three-dimensional rendering of the optic nerve head (ONH). (right) Shape- and sublayer-revealing cross-section of the ONH with optical coherence tomography (OCT). A wider and deeper physiological cup is an indication of ONH degeneration and apoptosis of nerve fibers.

ratio between cones and ganglion cells. Thus, each ganglion cell in the fovea receives input from just one cone bipolar cell, which in turn, is contacted, often, by just a single cone. The cone system, in other words, is not convergent. This almost 1:1 relationship of cones to ganglion cells serves to maximize visual acuity (high central resolution) at the center of the macula.

The way rods converge is strikingly different; here, the ratio between rods and ganglion cells is likely to be on the order of 100:1 or more. Each rod bipolar cell is contacted by several rods, and many rod bipolar cells contact a ganglion cell. Thus, there is a many:one relationship between rods and optic nerve fiber neurons, which can even reach 100:1. The response of the ganglion cell depends on the responses of the cells that feed into it, including the rods, the bipolar cells, and various lateral interconnections via horizontal cells and amacrine cells.

As we shall see in § 4.3.6, the pathways have a far more complicated structure that involves amacrine cells and ON and OFF types of rod bipolar cells. Rod ON signals flow from rod bipolar cell to AII amacrine cells to ON cone bipolar cells, while rod OFF signals are generated at synapses between AII amacrine cells and OFF cone bipolar cells.
resolution limit derives from the need to properly sample the signal. We can measure only the part of the input signal that falls on the sample positions, which are the cones, by including at least one unilluminated cone (minimum) between two illuminated cones (maximum).

Figure 4-56 (right) illustrates the Nyquist limit: The signal spatial detail (spacing between maximum and minimum) cannot be finer than the cone spacing. Simply put, to find the resolution limit of the human eye in relation to the discrete nature of the photoreceptor mosaic, we consider the average cone spacing of 2.6 μm, which corresponds to ≈ 0.5 arcmin ≈ 60 cycles/degree. At the digital/neural sampling limit, an image of an Airy disk peaking at one cone needs to be angularly sized with a radius ≈ 0.5 arcmin. This is, therefore, the ultimate limit to human vision, as was suggested as early as 1922.[408] Population studies have shown that the minimum angle of resolution in healthy emmetropic individuals is, indeed, about 0.6 to 0.7 arcmin.[269]

![Diagram of digital sampling](image)

**Figure 4-56:** Digital sampling (left) at more than the Nyquist limit (oversampling) and (right) at the Nyquist limit. The cone spacing produces a YES-NO-YES response to the projected sinusoidal function.

**Human eye minimum angle of resolution (digital/neural sampling limit)**

≈ 0.5 arcmin ≈ 60 cycles/degree.

![Image of letter E foveal cone sampling](image)

**Figure 4-57:** Letter E foveal cone sampling. (left) Sampling that corresponds to standard 20/20 visual acuity (1 arcmin MAR) and (right) sampling that corresponds to 20/10 visual acuity (0.5 arcmin MAR).

5.1 **Color in Optics**

To understand color vision, we need to understand the physics of light. The question of ‘color’ is a very interesting one. Color theory encompasses a multitude of definitions and concepts. **Color** is a human physiological perceptual property that we assign to objects or self-illuminating bodies, usually referred to as light sources, on the basis of spectral characteristics. In this sense, color is not only a property of electromagnetic radiation but is also an aspect of the observer’s visual perception. In the physical world of photons and electromagnetic waves, however, there are no colors with this familiar meaning. Physics, actually, only regards the different energies of photons that distinguish colors, or, under the wave theory, the different wavelengths or frequencies of the radiation.

A bundle of white light is just a mix of different wavelengths that can be decomposed into the constituent color components if it passes through a prism or water drops, forming a rainbow. Nature is not shy of demonstrating the beauty of colors. Spectra of colors can be observed by thin-film interference: This is essentially the reason we can see the incandescent colors in a soap bubble! The color at each point depends on whether the conditions for constructive interference exist for the specific optical path through the layer of water for that
lack of functional M-cone cells, although in deuteranomaly, there are two slightly different L-cone types; in other words, the M-cone responsivity (mutated, shown in gray in Figure 5-34 bottom center) is shifted toward the L-cone curve. There is no hard distinction, but instead a continuum, between the ‘-anopia’ and the ‘-anomaly’ types of dichromatic CVD, as these forms can range from almost complete loss of the respective channels to just mild deficiencies.

Figure 5-34: Cone responsivity for the major classifications of color vision deficiency. (top row) Normal color vision. (middle row) Dichromatic color vision deficiency. (bottom row) Anomalous trichromacy.

Note ❗: Color vision deficiency is NOT ‘color blindness.’ The term ‘color blindness’ is used today to describe only the most severe form of complete color vision loss and not most cases of CVD. And, no, protanopes and deuteranopes do not see red-green simply as shades of gray; this is a misconception. Individuals with red-green deficiencies confuse a host of reds, greens, browns, and oranges as various shades ranging from yellow to gray.

Figure 5-35: Normal color vision versus red-green and blue-yellow defects in the CIELAB color space.
Figure 5-46: In pseudoisochromatic testing, background and concealed pattern chromaticity coordinates are chosen near the locus positions so that the concealed pattern spots serve for both protan and deutan defects since their isochromatic confusion lines (fully presented in Figure 5-38) can be very similar.

The Ishihara test, originally published in 1906 by the Japanese ophthalmologist Shinobu Ishihara, was the first commercially available pseudoisochromatic test and helped to popularize the technique. Even today, many available pseudoisochromatic tests refer to an ‘Ishihara compatibility’ and claim to be evolutions of this test. The Ishihara test is considered the benchmark for the rapid screening of red-green color deficiencies.

The test consists of several plates (depending on the version, the test may contain 38 plates, 24 plates or 14 plates), each of which has a circle filled with dots of random color and size. The concealed pattern is a numeral (single or double-digit) or a path-shape, forming vanishing and transformation designs for red-green defects. The examinee is asked to name/identify the number shown or follow the concealed pattern using a fingertip.

Figure 5-47: Ishihara test plates. (left) Demonstration plate: The numeral shown is viewable by all, including the red-green deficient. The second plate is vanishing design (reading the numeral 6; if you read the numeral 5, you may have red-green deficiency). The third plate is transformation design (reading 74 here and not 21). (right) The fourth design is a blue line to most observers. (The colors in these images as seen on this page/screen might not be reproduced accurately due to differences in the reflectance from the printed surface and the viewing illuminance compared to those intended for the exam.)

The notion that all rays from an object point meet at a single common image space ‘point’ belongs to the realm of geometrical approximation. Geometrical optics, indeed, employs terms such as the focal or image ‘point.’ A point is, simply, a mathematical construct, not a physical entity. A point has no dimension, volume, surface area, or length. In reality, and specifically, as it applies to optics, the concept of the point is used to denote a very small surface area.

Although the focal point is a conceptual construct, physically it corresponds to a constricted, physical distribution of light, called the blur circle. Even in geometrical optics (in other words, ignoring diffraction), there is always some blur. We may call it an image ‘point,’ if all rays from an object converge to what may be considered a very small surface area and cross-section.

Figure 6-1: Under increasing magnification, the ‘point’ of convergence reveals that the ‘focus’ point is not a point, but has a small, specific surface area. The ‘point’ is not a point!
Example ☞: If the distal point has −1.0 D vergence difference from the conjugate object point, the proximal point has +1.0 D vergence difference from the conjugate object point. Then the dioptric depth of field is reported as 2.0 D or ±1.0 D. Note that the linear depth of field is not symmetrical with respect to the conjugate point since distances are reciprocal and not proportional to vergence.

Example ☞: Rigoletto is holding a lens in the air. The distal limiting position has −2.0 D vergence, and the proximal limiting position has −8.0 D vergence. Estimate the dioptric and the linear depth of field.

\[ \text{The dioptric DoF is } |(-2.0 \text{ D}) - (-8.0 \text{ D})| = 6.0 \text{ D or } \pm 3.0 \text{ D.} \]

The conjugate point is at −5.0 D vergence (the dioptric midpoint between the distal and proximal limiting points, spaced by ±3.0 D); therefore, its location is \( \frac{1}{-5.0 \text{ D}} = -20 \text{ cm} \) in front of (before) the lens.

The distal point, with −2.0 D vergence, is at \( \frac{1}{-2.0 \text{ D}} = -50 \text{ cm} \) in front of the lens, and the proximal point, with −8.0 D vergence, at \( \frac{1}{-8.0 \text{ D}} = -12.5 \text{ cm} \) in front of the lens. The linear depth of field of Rigoletto’s lens is the interval between the two limiting points (50 cm – 12.5 cm) = 37.5 cm.

The corresponding expanse in image space (inside the eye) is the depth of focus. In photography, the depth of focus is the range in which the sensor (or the film) can be moved back and forth with respect to the lens so as to maintain a manifest change in image sharpness. In the eye, the depth of focus is the range of perceptual tolerance to out-of-focus images; within the depth of focus, small differences in image sharpness are not noticeable.

\[ \text{Figure 6-12: Example of linear depth of field calculation when the dioptric depth of field is 6.0 D.} \]

\[ \text{Figure 6-13: The depth of focus is the perceptual tolerance of the retinal defocus. Any possible sharpness difference for images formed within the depth of focus is not noticeable. See also Figure 3-37.} \]
6.6 SPECTACLE CORRECTION OF MYOPIA AND HYPEROPIA

6.6.1 Correction of Myopia: Optical Principles

Let us see what it takes to correct myopia. Regardless of the type (refractive/axial), the image of a distant object (collimated beam, vergence 0.0 D) is formed before (in front of) the retina. The purpose of the correction is to bring (shift) the focal point of the system 'Eye + Correction' exactly to the retina. Using the terminology of the far and near points, in a myopic eye, the far point is at a finite distance in front of the eye. Equivalently, thus, we want to shift the far point of the system 'Eye + Correction' to infinity. Consider a far point at 2 m (noted as −2 m because it is to the left of the eye, against light propagation). By definition, this eye has −0.50 D of myopia, as the far-point vergence is

\[
\text{far-point vergence} = \frac{1}{-2 \text{ m}} = -0.5 \text{ D}
\] (6.8)

How can we correct this eye? This eye can clearly see (without accommodation) only those objects that are located just 2 m in front of it, not distant objects at optical infinity. The solution is simple: Bring the image of a distant object to where the eye can see it—to its far point.

This can be achieved with a corrective lens whose optical power \( F_{\text{corr}} \) is such that, when presented with an object at infinity (vergence = \( 1/\infty = 0 \) D), it forms an image 2 m to its left. Such a lens is a minus lens of focal length \( f' = -2 \) m, or of optical power \( F_{\text{corr}} = -0.5 \) D. The secondary focal point of this corrective lens is placed at the far point of the myopic eye.

![Figure 6-56: A minus lens with a focal length equal to the far distance of a myopic eye brings an image from optical infinity to the far point.](image)

The image formed by the minus lens is now the object observed by the eye, located at the far point of the myopic eye. The image of this object is formed sharply on the retina. Now, the system 'Eye + Correction' performs optically like an emmetropic eye.
7.1 THE NATURE OF ACCOMMODATION

7.1.1 The Need to Adapt to Shorter Distances

Naturally, the eye is set to observe objects at far distances; the emmetropic eye has the optical power to form a sharp retinal image from an object situated at a far distance, which, in the optics of the eye, refers to any location 6 m or more away. On the other hand, when we are reading, writing, or texting, the objects are not at a far distance but instead are close, at a short distance, usually at arm’s length. The closest distance to which an object can be brought and still be optically conjugate to the retina is the near point (§ 6.1.3). A typical value for the near point is 25 cm, although it varies significantly with age, status of eye ametropia, and other factors.

To bring close objects into focus, the eye needs accommodation, defined as the ability of the eye to make the retina optically conjugate to a range of target distances.\footnote{Duke-Elder S, Abrams D. System of Ophthalmology. in: Ophthalmic Optics and Refraction, S. Duke-Elder, ed. Henry Kimpton, London, 1970. Vol. V:156-7.} When the object is located at the near point, accommodation is maximum. When the object is located at the far point (§ 6.1.3), accommodation is zero, and the state of the eye is called relaxed, unaccommodated, or disaccommodated.
We convert this distance to meters. Also, we use the minus sign to indicate that the object is to the left. Therefore, object location is \( x = -0.66 \) m. Then, vergence is \( L_x = 1/(-0.66 \text{ m}) = -1.50 \text{ D} \).

Example \( \Rightarrow \): Odysseus, an emmetrope (\( \therefore \) far point at infinity), is reading a scroll at arm’s length (66.6 cm). What is the magnitude of accommodation \( A \)?

At that fixation point, vergence is \( 1/(-0.66 \text{ m}) = -1.50 \text{ D} \). Odysseus has a far point at infinity, so the far-point vergence = 0.0 D. The magnitude of accommodation is the difference between the far-point vergence (0 D) and the fixation-point vergence (-1.50 D): (0.0 D) – (-1.50 D) = +1.50 D.

**Figure 7-7: Emmetropic eye and an object 66 cm in front of it = accommodative demand +1.50 D.**

### 7.2.2 Amplitude of Accommodation

The closest an object can be brought to the eye and still be clearly viewed is the near point. At this near point, an individual exerts the maximum accommodative response, whose magnitude is the **amplitude of accommodation** (\( AoA \)).

\[
\text{Amplitude of accommodation} = (\text{far-point vergence}) - (\text{near-point vergence}) \quad (7.2)
\]

Example \( \Rightarrow \): Odysseus’ emmetropic eye (\( \therefore \) far point at infinity, \( L_{FP} = 0.0 \text{ D} \)) has a near point at 25 cm in front of it (\( L_{NP} = -4.00 \text{ D} \)). The amplitude of accommodation is \( AoA = +4.00 \text{ D} \):

\[
AoA = L_{FP} - L_{NP} = \frac{1}{\infty} - \left(\frac{1}{-0.25 \text{ m}}\right) = 0.00 \text{ D} - (-4.00 \text{ D}) = +4.00 \text{ D}
\]

**Figure 7-8: Amplitude of accommodation for an emmetropic eye.**

Example \( \Rightarrow \): Achilles, a \(-6.00 \text{ D} \) myope (\( \therefore \) far point at 16.6 cm in front of the eye), has a near point at 10 cm. What is his amplitude of accommodation when he is not wearing any spectacles or contact lenses?
2 m in front of the lens, its location is 2.015 m in front of the eye (after adding the vertex distance). Thus, the vergence of the fixation point that reaches the eye is \( \frac{1}{-2.015} \) m = −0.496 D = −0.50 D.

The above steps lead to the calculation of the vergence of the fixation point. Now we can proceed with the calculation of the magnitude of accommodation using relationship (7.1):

\[
\text{Magnitude of accommodation} = \text{far-point vergence} - \text{fixation-point vergence}
\]

Spectacle-lens-corrected hyperope: \( = +4.00 \text{ D} - (-0.50 \text{ D}) = +4.50 \text{ D} \).

**Figure 7-16: Accommodative demand for a spectacle-plane corrected hyperope.**

Compare this result to +4.00 D required if that eye is corrected with contact lenses; the spectacled hyperope requires more accommodation: +4.50 D. Another way to say this is the contact-lens-corrected hyperope requires less accommodation.

**Figure 7-17: In a hyperopic eye, as the vertex distance decreases, the accommodative demand for the same eye-object distance decreases. A hyperope, when viewing a near object, accommodates less when the ametropia is corrected with contact lenses than when it is corrected with spectacles. Compare with Figure 7-24.**
Example ☞: Eurycleia is a myope with Rx $-2.57$ D at $11.5$ mm vertex distance. The equivalent contact lens Rx (0 mm vertex) is $-2.50$ D. She is hand-spinning wool $40$ cm in front of her eyes. How much less does she converge when observing the spindle using her spectacles if her interpupillary distance is $70$ mm?

First, we calculate convergence $[\Delta]$ when she views the spindle at the same distance (this involves several assumptions regarding the presence or lack of accommodative response) without any correction. We use Eq. (7.4): convergence $[\Delta] = \text{IPD [cm]} \cdot 100 \div \text{distance [cm]}$. Here IPD $= 7$ cm, distance from the corneal plane $= 40$ cm, i.e., distance from the CoR $= 41.35$ cm. Convergence $= 7 \cdot 100/(41.35) = 16.92^\Delta$.

When wearing the contact lens ($-2.50$ D), the spindle is located $40$ cm from the contact lens; the image of the spindle is formed $20$ cm from the corneal plane and $21.35$ cm from the CoR. Thus, convergence with contact lenses $= 7 \cdot 100/(21.35) = 32.78^\Delta$.

When wearing the spectacle lens ($-2.57$ D at $11.5$ vertex), the spindle is located $38.85$ cm from the spectacle lens; the image is formed $19.36$ cm in front of the lens (verify with imaging relationships), which is $20.51$ from the corneal plane and $21.8$ cm from the CoR. Thus, convergence with spectacle lenses $= 7 \cdot 100/(21.8) = 32.11^\Delta$. Eurycleia has a slight advantage of about $0.67^\Delta$ using a spectacle lens.

Figure 7-31: Eurycleia convergence (left) Contact-lens correction and (right) Spectacle-lens correction.

There is an additional aspect [discussed in § 10.4 as Prentice’s rule, relationship (10.12)] when using spectacles because the eyes turn to gaze while the spectacles do not turn, but are fixed at $13.5 + 11.5$ mm $= 25$ mm from the center of rotation of the eyes (recall that the CoR of the eye is $13.5$ mm inside the eye). Using simple similar-triangle geometry, we can infer that the line of sight of each eye intersects the lens at $0.211$ cm inward, which is the equivalent of decentering each lens by that amount outward (temporally).

This induces a base-in prism with the $-2.50$ D lenses. We use the Prentice rule to calculate the induced prism: $P[\Delta] = -2.50$ D $\cdot 0.211$ cm $= -0.52^\Delta$ per eye.
CHAPTER 8
ASTIGMATISM

8.1 GEOMETRY AND CLASSIFICATION OF ASTIGMATISM

An implicit assumption up to now has been that the corneal surface is part of a perfect sphere. Regardless of the cross-section orientation, the cornea is assumed to produce circular cross-sections: If we imagine that we intersect the cornea along the horizontal, the vertical, or any other oblique orientation, the cross-section yields circular surfaces of equal, fixed radii of curvature.

These radii of curvature are generally smaller in myopia (steeper cornea) and larger (flatter cornea) in hyperopia. In any case (either emmetropia, myopia, or hyperopia), the points of focus corresponding to any two individual corneal cross-sections coincide—just not always on the retina.

*Figure 8-1: Ray convergence along two distinct meridians for an emmetropic, myopic, and hyperopic eye. The points of focus corresponding to any two cross-sections coincide.*
**Figure 8-11:** The four illustrative representations of (top) with-the-rule astigmatism and (bottom) against-the-rule astigmatism: shape of the football, meridians, lens cross, and bow-tie pattern in corneal topography.

**Figure 8-12:** Simple myopic astigmatism (top) with-the-rule and (bottom) against-the-rule. The two forms differ by a rotation of the lens cross (and the corresponding focal lines defining the Sturm conoid) by 90°.

Food for thought 🧠:
The distinction between myopic, hyperopic, and mixed type relates to the axial placement of the focal lines (Sturm conoid) with respect to the retina. Both examples in Figure 8-12 are simple myopic. If we rotate the lens cross by 90°, they are still the same in terms of refractive power, but are of the opposite-rule type.

Note📝: Astigmatism cannot be classified according to the criterion of axial length—there is no axial astigmatism. This is because the axial length in each eye has a single value, the same for the two principal meridians, so the axial length in one eye cannot be longer or shorter than in the other eye.

A third distinction of astigmatism is based on symmetry. If the principal meridians are perpendicular to each other (90°), then astigmatism is regular or symmetric. If the principal meridians deviate from being at right angles, we have an irregular or asymmetric astigmatism.
Note: This formula for spherical equivalent works with either plus or minus Rx prescription form. It is, however, important to keep tabs on the algebraic signs!

Practice examples with answers:

Case A: Sancho Panza’s Rx is $-2.50 \times 180$; SE is $-2.50 + \frac{1}{2} \cdot (-1.00) = -2.50 + (-0.50) = -3.00$ D.
Case B: Dapple’s Rx is $-2.00 + 2.00 \times 090$; SE is $-2.00 + \frac{1}{2} \cdot (+2.00) = -2.00 + (+1.00) = -1.00$ D.

Note: The spherical equivalent is independent of the choice of minus or plus Rx notations.
Sancho Panza’s plus-cylinder Rx: $-3.50 + 1.00 \times 090$; SE: $\frac{1}{2} \cdot [-2.50 + (-3.50)] = -3.00$ D.
Dapple’s minus-cylinder Rx: $0.00 - 2.00 \times 180$; SE: $0.00 + \frac{1}{2} \cdot (-2.00) = 0.00 + (-1.00) = -1.00$ D.

Note: SE can also be computed by the average of the two spherical powers in the lens cross.
Sancho Panza’s lens cross: $-2.50 @ 180^\circ / -3.50 @ 90^\circ$; SE: $\frac{1}{2} \cdot [-2.50 + (-3.50)] = -3.00$ D.
Dapple’s lens cross: (plano) @ 180° / -2.00 @ 90°; SE: $\frac{1}{2} \cdot [0.00 + (-2.00)] = -1.00$ D.

In a trial lens or phoropter refraction examination, if we simply increase the cylinder in the typical spherocylindrical formulation, the spherical equivalent is affected: A half sphere must be compensated (e.g., +0.25 D) per each cylinder diopter change (e.g., −0.50 D) to keep the placement of the circle of least confusion on the retina.

Example: During the phoropter test, we place a $-2.00 -1.50 \times 180$ in front of Marcela’s eye. Subsequently, during the examination, the cylinder changes to $-0.50 \times 180$. What should the new spherical power be to keep the placement of the circle of least confusion on the retina?

Cylinder power changes from $-1.50 \times 180$ to $-0.50 \times 180$; this means that the change in cylinder power is $+1.00$ D. We therefore need to compensate by adding $-0.50$ D sphere, making the new sphere power $-2.50$ D.

Note: Mixed astigmatism usually has a (near) zero SE. Zero occurs for equally mixed astigmatism only.

Example: Ambrosio’s Rx is $-1.00 + 2.00 \times 090$. SE = $-1.00 + \frac{1}{2} \cdot (+2.00) = -1.00 + (+1.00) = 0.00$ D.
Lens cross: $-1.00 @ 90^\circ / +1.00 @ 180^\circ$; SE (average of the two powers) = $\frac{1}{2} \cdot [-1.00 + (+1.00)] = 0.00$ D.
This is equally mixed astigmatism.

Example: Grisóstomo’s Rx is $-0.75 + 1.25 \times 180$. SE = $-0.75 + \frac{1}{2} \cdot (+1.25) = -0.75 + 0.625 = -0.125$ D.
Lens cross: $-0.75 @ 180^\circ / +0.50 @ 90^\circ$; SE (average of the two powers) = $\frac{1}{2} \cdot [-0.75 + (+0.50)] = -0.125$ D.
This is mixed but not equally mixed astigmatism.
While cylinder magnitude differences of up to 1.50 D are not uncommon, cylinder magnitude differences of 3.00 D and greater are quite rare. Again, this example serves to illustrate *in extremis* the problem with fully correcting different magnitudes of astigmatism: The corrected monocular images are disproportionally distorted, and the resultant binocular image perception leads to depth perception alterations.

![Figure 8-54: Uncorrected and fully corrected image perception for OD eye –1.00 –0.50×180 and OS eye –1.00 –3.00×180—the case of same-axis (here 180) but-different-magnitude WTR astigmatism. This example is further discussed in § 9.3.1, where we learn that retinal image minification is 2.25% for OD and 6.00% for OS.]

**Clinical Pearl 🤔**: Where should the best astigmatic correction be applied?

The obvious answer would be as close as possible to the seat of astigmatism. If the seat of astigmatism and the surface upon which the correction is applied are spaced by some distance, the full correction will still result in residual, secondary astigmatism, which is a high-order aberration.

Since secondary astigmatism is a high-order aberration (these concepts are presented in *Ocular Imaging*, Volume 5 of this series), residual amounts can be present even if the magnitude of the astigmatic correction is calculated correctly per the sphero-cylindrical part, which corresponds to a low-order aberration.

If the astigmatism is corneal (which is the most prevalent type; significant lenticular astigmatism does occur, but rarely), the optimal solution is laser refractive surgery or contact lenses. If spectacle or intraocular lens correction is selected, the larger the distance of the correcting lens from the cornea (vertex distance/effective lens position) the larger the residual secondary astigmatism.
9.1 **SOME HISTORICAL FACTS**

Vision-correcting glasses are one of humankind’s most important inventions. We do not know exactly who to thank for their invention. Several individuals claim to have invented spectacles, but the definitive historical evidence is clouded by unfounded information.  

Due to the highly technical nature of crafting spectacles, their invention likely happened in northern Italy sometime during the 14th century, either in Veneto or Tuscany; in these regions, at the time, intricate glassmaking was elevated to unparalleled levels. The earliest depiction of spectacle glasses in art dates back to 1352 and was found in a fresco created by Tomaso Barisini (better known as Tommaso da Modena) in the Chapter House of the Seminario, which is attached to the Basilica San Nicolò of the Treviso Cathedral (Veneto, Italy).  

It’s possible that the invention of spectacles can be credited to either Salvino D’Armato or the Dominican friar Alessandro della Spina.  

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901 Ronchi V. Perche non si ritrova l’inventore degli Occhiale? Rivista di Oftalmologia. 1946; (1):140.  
In the simple approach introduced in § 6.6.1 (correction of myopia), the power of a corrective lens equals the degree of myopia/hyperopia. However, this is an approximation, albeit a very good one, given that this is the logic behind contact lenses (§ 9.3) and refractive laser surgeries, since, in both of these methods, the lens ‘touches’ the eye. Spectacle lenses do not touch the eye, but sit at a vertex distance, usually, of 12 to 15 mm. This distance is affected by the globe and orbit shape (e.g., deep-set eyes have a larger vertex) and by the shape of the mounting frame. Contact lenses are prescribed with zero vertex distance.

Here is an example of a corrective lens with optical power +10.00 D, placed at a distance of 15 mm in front of the eye (Figure 9-13). A collimated beam (initial vergence 0 D) just after the lens obtains vergence of +10.00 D. To correct the +10.00 hyperopia, the rays need to converge at the far point $M_R$, which is also the focal point of the lens $F' = SM_R = 100$ mm.

For a lens to correct the +10.00 hyperopia when in contact with the cornea, i.e., at point C, the effective power must be the reciprocal of distance $CM_R = 100 - 15$ mm = 85 mm (the far point of the eye, $M_R$, is the same!) The power of that lens in contact with the eye is the effective downstream power $F_d = 1/(0.085 \text{ m}) = +11.76$ D.

In a myopic eye (Figure 9-14), the corrective spectacle lens has power −10.00 D, and the distance to focus is $CM_R = -100 + (-15)$ mm = −115 mm. For a lens to correct the −10.00 D when in contact with the cornea, i.e., at point C, the optical power is $F_d = 1/(-0.115 \text{ m}) = -8.70$ D.

In general, for a lens to have the same convergence effect when transposed (shifted) to position $d$, the effective focal length at the new location $f_d$ must be

$$f_d = f_o - d$$  \hspace{1cm} (9.6)

where $f_o$ is the focal length of the lens in the initial position, and $d$ is the distance to the new position. This relationship, when presented in terms of optical power, takes the form

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**Figure 9-13:** A plus-powered corrective lens at 15 mm vertex distance.

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OPHTHALMIC LENS OPTICS
10.1 Thin Prism Optics and Geometry

10.1.1 Prism Deviation Angle

A prism is a transparent optical element with two non-parallel refracting sides that form the apical angle \( A \). A prism deviates rays without affecting their vergence; a collimated beam of light emerges collimated but in a different direction. The more powerful the prism the larger the deviation angle \( \theta_E \) formed between the emerging ray and the ray that is incident on the first surface (\( \theta_i \)). If the angle of refraction at the second interface is \( \theta_t \), then (see Figure 10-1):

\[
\theta_E = \theta_i + \theta_t - A
\]

(10.1)

The deviation angle is dependent on the prism’s refractive index (which is dependent on the incident light wavelength and the prism material), the apical angle (prism geometry), and the angle of the incident ray. This can be particularly complicated, especially if the apical angle is greater than 30° (what we call a large angle).\(^938\) To accurately calculate the deviation angle, we need the details about four angles: the incident angle \( \theta_i \) and refraction angle \( \theta_t \) at interface 1, and the incident angle \( \theta_i \) and refraction angle \( \theta_t \) at interface 2, as shown in Figure 10-1. In

\(^{938}\) *Introduction to Optics* § 3.3.2 Minimum Angle of Deviation.
Figure 10-8: Angle $\theta_E$ due to prism deviation and angle $\theta_{EFF}$ entering the eye when object is nearby.

$\theta_{EFF} = \theta_E - \theta_c \Rightarrow P_{EFF}/100 = (P/100) - (h/PN)$ and $P_{EFF}/100 = h/PC \Rightarrow h = PC \cdot P_{EFF}/100$

Combination of the above yields: $P_{EFF}/100 = P/100 - (PC/PN) \cdot P_{EFF}/100 \Rightarrow$

Effective Prism Power:

$$P_{EFF}[\Delta] = \frac{P[\Delta]}{1 + \frac{PC}{PN}}$$  

(10.8)

Example ☞: What is the effective prism power for a prescription prism of $P = 3.0 \Delta$, set at vertex distance 11.5 mm, when Aeneas views a near object at 25 cm from his eye (the distance from the corneal plane)?

Figure 10-9: Calculation of effective prism power in the presence of a near object.

Step one is to determine the distance PC from the prism to the center of rotation of the eye. In an average-sized eye, the center of rotation is 13.5 mm behind the anterior pole of the cornea ($\S$ 2.4.3). Thus, the distance from prism to the center of rotation is $11.5 + 13.5 = 25$ mm.

Step two is to determine the distance PN from the prism to the near object. Since the object is 250 mm from ‘the eye,’ it is $250 - 11.5 = 238.5$ mm from the prism (all distances use the same distance unit, millimeters).
Examples ☞: The following Rx’s produce vertical imbalance:
+2.00 OD / +4.00 OS; the difference of the magnitudes of the two prisms.
+2.00 OD / –1.00 OS; the sum of the magnitudes of the two prisms.
–1.00 +2.00×180 OD / –1.00 –1.00×180 OS; the sum of the magnitudes of two prisms (draw the lens cross!)

Example ☞: Othello’s Rx is OD –6.00 D / OS –2.00 D. What is the induced vertical imbalance when his gaze shifts 5 mm downward from the lens optical centers?

![Diagram of vertical imbalance in Othello’s eyes]

Figure 10-38: Vertical imbalance in Othello’s eyes. The downward gaze shift is simulated with an upward lens movement.

We calculate separately the decentration-induced prism due to Othello looking downward by 0.5 cm. A downward shift in the gaze is equivalent to an upward lens decentration by the same amount.

OD prism is (–6.00 D) · (0.5 cm) = –3.0^A, or 3.0^A BD. OS prism is (–2.00 D) · (0.5 cm) = –1.0^A, or 1.0^A BD. The vertical imbalance is 2.0^A, reported on the most-affected eye as 2.0^A BD OD.

Vertical Imbalance Practice examples with answers ☞:

Valeria’s Rx is OD –2.00 / OS +1.00. Her gaze shifts downward by 1 cm. Vertical imbalance is 3.0^A BD OD.

Miranda’s Rx is OD –3.00 –2.00×180 / OS –2.00 –1.00×180. When her eyes shift downward by 3 mm, a 0.6^A BD vertical imbalance is induced, noted on the most affected OD eye (hint: draw the lens cross!).

When Prospero gazes downward 5 mm, a 1.5^A BD vertical imbalance is noted in his OS eye. What is the cylinder in his OS eye if the Rx is OD –3.00 +2.00×180 and the sphere in OS is –4.00 D? (Answer: plano.)

Reynaldo’s Rx is OD +3.00 –2.00×180 / OS –2.00 –2.00×090. His gaze shifts downward by 5 mm through the optical centers of the lenses. Vertical imbalance induced is 1.5^A BD noted on the most affected OS eye.

Note ☞: In some textbooks, vertical imbalance is reported on the most affected eye as the prism power required to compensate for this imbalance. In this case, the prism direction is opposite to what was introduced here. Attention is therefore warranted to the proper definition of ‘imbalance.’
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