Chapter 1
Introduction to the Ponderomotion Processes and Overview of Related Phenomena

It was beyond any understanding and a shocking surprise when Roland Sauerbrey (1996) measured ultrahigh acceleration of macroscopic objects having densities in the range of the solid state. These accelerations were over 10,000 times higher than ever measured before in a laboratory. His convincing measurement at laser interaction with a target was easily seen by the Doppler blue shift of spectral lines in the reflected light. Sauerbrey's mention of this most extraordinary observation was hidden in the abstract of a long paper. Why was no exceptional attention given to this fact? One problem was that extremely high-quality lasers were needed, as discovered and clarified by Jie Zhang (1998); see the third paragraph of Chapter 8.

The acceleration was on the order of a billion billion times higher than Earth's gravitational acceleration $\boldsymbol{g}$. It was well known from studies of laser interaction with targets that very high accelerations of dense blocks of materials could be produced by the largest laser on Earth at the National Ignition Facility (NIF) in Livermore, California. It could heat a metal surface and ablate it from the generated plasma by an acceleration of a thousand billion times faster than $\boldsymbol{g}$. This remarkable result was achieved with very short laser pulses of one thousandth of a millionth of a second (a nanosecond, or ns).

However, with one thousand times shorter high-power pulses, a different mechanism was observed. This new situation appeared following the development of the discovery by Gerard Mourou and associates (Strickland et al. 1985; Mourou 1994; Perry et al. 1994; Mourou et al. 1998; Mourou et al. 2013) that petawatt (PW) lasers could be produced with a picosecond (ps) pulse duration, more than one thousand times shorter than nanosecond pulses. Today, the pulses produced are even shorter and are called attosecond pulses (10^{-18} s, see Krausz et al. 2009).

With shorter pulse durations, the interaction with nanosecond pulses, i.e., the heating and gas dynamic pressure processes of the thermal ablation, could be ignored. In contrast, laser pulse energy would be nonthermally and directly converted into mechanical energy of the ablated dense plasma blocks.

The way to these results was paved by a large succession of advancements in physics that are the focus of this book. Newton's discovery of the gravitational force that determines the mechanical motion of masses led to the understanding of Kepler's discovery of the kinematics behind precisely elliptic planetary motion. Coulomb's discovery of the electric force between electric charges was, in turn, a crucial consequence of Newton's second law, which formulated force. But a next important step was the discovery by William Thomson (1845)—later Lord Kelvin—of how electrically uncharged bodies can be moved by electric fields with what is called a ponderomotive force. This is a nonlinear force formulated as a quadratic expression of the force quantities of electrostatic fields. Indeed, James Clerk Maxwell would later combine these "nonlinear forces" with magnetic and temporal nonconstant processes using the stress tensor in his formulation of electrodynamics.

The main advantage of the extremely fast laser–plasma interaction for ignition of controlled nuclear fusion (see Chapters 9 and 10) is that it allows one to overcome the enormous problems of complex systems. This was discovered and treated by Robert May (1972; 2011)—now Lord May of Oxford—in his work on the physics of thermal processes, in contrast to the microscopic world of atoms as it was also formulated in 1952 by Edward Teller (2001). The pioneering work of Lord May has led to the universal application of these solutions not only in physics, but also in zoology as applied to animal populations, in medicine as applied to infectious diseases, and in economics as applied to banking systems and the financial crisis (Haldrane et al. 2011).

This opportunity to reduce complex problems by extremely short-time interactions is the special advantage of the new scheme specifically presented here. Lasers permit the driving of reactions through side-on ignition of fusion of uncompressed nuclear fusion fuel at very short times using the nonlinear force acceleration of plasma blocks. Teller (2001) summarizes the problem in the following way:

> Research on controlled fusion means dealing with the hydrodynamics of a plasma. I had a thorough respect for the fearsome nature of hydrodynamics, where every little volume does its own thing. Plasma does not consist of molecules, like a gas, but of ions—heavy slow moving positive ions—and light, fast moving electrons those, in turn,

> create and are coupled with electric and magnetic fields. For each little volume of plasma, several questions have to be answered: How many positive ions? How many electrons? How fast does each move on the average? What is the electric force, and what is the magnetic force acting on them?
>
> Mathematicians can predict the flow of matter as the volumes involved move in an orderly way. But even hydrodynamics of air was, and to some extend is (see weather forecasts), beyond the grasp of mathematics. Theoreticians of the nineteenth century proved that flying was impossible! In the twentieth century, they retreated to the statement that flying is impossible unless the air flow is confused and disordered (turbulent). Hydrodynamics as a science remains uncharted water.
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> The same complications occur in planning a thermonuclear explosion. But an explosion occurs in a so short time that many of the complicated phenomena have no chance to develop. Even so, it took a decade from Fermi's first suggestion of a thermonuclear reaction to the point (which occurred after the first full-scale demonstration of fusion) that the theoretical calculations of the explosions were reasonably complete. I had no doubt that demonstrating controlled fusion would be even more difficult (Teller 2001: see page 344).

Sixty years later, this problem with plasma physics and hydrodynamics is still not fully solved, although much has been learned. Only deep and rigorous research can lead us forward, as seen from the eminent achievements of Lord May of Oxford, who ingeniously changed the 19th-century initial insufficient approaches in theoretical physics in order to now master complex systems. His questions were, "will a large complex system be stable?" (May 1972) and will it seek to consider "systemic risk in banking systems"? (Haldrane et al. 2011).

The new situation for laser fusion as presented here is, as Teller mentioned, that the interaction of petawatt–picosecond laser pulses is fast enough to avoid the complications that usually appear. This is why this advance was considered revolutionary; as Steven Haan said in an interview to the Royal Society of Chemistry (referring to the NIF laser for fusion at the Lawrence Livermore National Laboratory): "This has the potential to be the best route to fusion energy" (Haan 2010).

Putting these general aspects aside, the questions of Kelvin's ponderomotive force highly influenced key developments of laser physics and the exploration of extraordinary new applications for forces in irradiated targets. These applications include the generation of clean, aneutronic nuclear fusion for energy generation, where even the problems of intermediary neutron generation are avoided (Tahir et al. 1997) to eliminate the still existing problem connected with the usual fusion of heavy and superheavy hydrogen, deuterium, and tritium DT.

The introductory overview of this chapter should explain in an abbreviated way the historical background of the ponderomotive force—extensively discussed in the 19th century—which has nearly been forgotten since then, until laser-produced plasma required a reconsideration. This renaissance included many conscious or unconscious misunderstandings and incorrect expressions; therefore, we are sketching here first some of the crucial points in order to explain that a much more general description is necessary for the electrodynamic forces on single particles or plasmas produced by the laser fields, which sometimes have extremely high values and are sometimes of durations in the range of femtoseconds.

Furthermore, there will be a rough description of some phenomena that are interesting in laser–plasma interaction and are related to the electrodynamic forces, which, since their introduction, have been called "nonlinear forces." This refers to the discussion in the following chapters that are more general, including the special ponderomotive forces depending not only on static, but on time-averaged high-frequency, stationary or transient fields with or without dissipation (thermalizing by collisional absorption). Indeed, this all is restricted to plasmas including their properties of dispersion.

The ponderomotive force was discovered by Kelvin (1845). It is a force density f_K produced in a dielectric medium with a refractive index n ($n^2 = \varepsilon$ is the dielectric constant), causing a polarization **P**, given by the scalar product with the tensor $\nabla\mathbf{E}$ (where **E** is the electric field)

$$f_K = \mathbf{P}\cdot\nabla\mathbf{E} \text{ (in SI units)} = \mathbf{P}\cdot\nabla\mathbf{E}/(4\pi) \text{ (in cgs units)}. \tag{1.1}$$

The vector symbols are used in the following way. The scalar product between two vectors **a** and **b** is given by $\mathbf{a}\cdot\mathbf{b}$, the vector product is given by $\mathbf{a}\times\mathbf{b}$, and the dyadic (undefined) product leading to the tensor **ab** is given without any sign between the vectors.

In order to drastically underline the controversial (or at least complex) situation; one referee of a leading journal argued that Eq. (1.1) contains all that is going to be discussed in this book, and none of this needs to be published because it is not new and everything is known about Eq. (1.1). I plan to convince the reader that there is more to be said, at least for plasmas and electrons.

Kelvin's formulation, Eq. (1.1), should now be rewritten. Remembering the definition of polarization,

$$\mathbf{P} = (n^2 - 1)\mathbf{E}/(4\pi), \tag{1.2}$$

Eq. (1.1) can be rewritten as

$$f_K = \frac{1}{4\pi}\left(\frac{n^2 - 1}{2}\right)\nabla\mathbf{E}^2 - \frac{1}{4\pi}(n^2 - 1)\mathbf{E}\times(\nabla\times\mathbf{E}). \tag{1.3}$$

In electrostatics one can express the electric field **E** by the gradient of a potential ψ, indicating that this is free of curls:

$$\mathbf{E} = -\nabla\psi; \qquad \nabla\times\mathbf{E} = 0. \tag{1.4}$$

This means that the last term in Eq. (1.3) is zero in electrostatics and one simply has the ponderomotive force expressed by the gradient of $\mathbf{E}^2$, the well-known *electrostriction*, or *electrostrictive force*. One may note that in the case that the rotation of **E** (i.e., $\nabla\times\mathbf{E}$) is different from zero as given for nonelectrostatic cases or for the high-frequency case by Maxwell's equations, even Kelvin's historic ponderomotive force is more complicated than simply the gradient of the square of the electric field.

One may remember the following cases of the classical electrostatic ponderomotive force of Kelvin, Eq. (1.1). Fig. 1.1(a) shows the Coulomb force acting on a charged particle within the homogeneous field of two condenser plates. If in Fig. 1.1(b) the charged particle is substituted by a sheet of electrically uncharged dielectric material, no force will act on it since there is no excess charge and the tensor $\nabla\mathbf{E}$ for a force [Eq. (1.1)] is zero. In Fig. 1.1(c) we consider a metallic sphere with positive charge producing a radial electric field in the air. If a dielectric material with $\varepsilon > 1$ is within this electric field, it will be pushed by the ponderomotive force [first term of Eq. (1.3)] towards the metal, i.e., towards increasing $\mathbf{E}^2$ by electrostriction, in the absence of any Coulomb force.

If the dielectric constant of the material is less than that of the surrounding medium, i.e., if there is a liquid with high dielectric constant (Fig. 1.2), then the material will be pushed towards lower values of $\mathbf{E}^2$. If there are more positively charged metal spheres (Fig. 1.2) or if there is a tetrahedral geometry, the electric field will result in a minimum between the spheres, and the material will then be pushed into the minimum and perhaps be squeezed together with another material of the same lower dielectric constant than the surrounding liquid. This is an important technique for merging living cells and forcing them to take on improved genetic properties or for studying cancer problems (Coster et al. 1995).

Being aware of the restrictions when mixing electrostatics with high-frequency fields (limited strictly to nontransient time-averaged cases), we now follow a consideration to see the special case of the *ponderomotive potential*. We expressively note that this is a very special limitation. For plasma

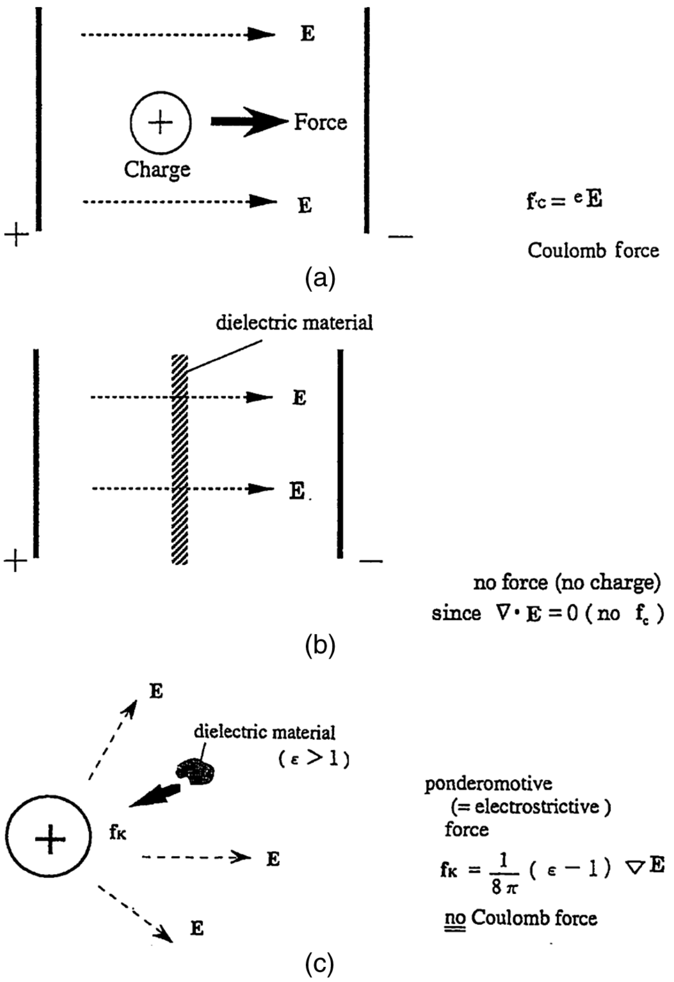

Figure 1.1 Electrostatic forces (Coulomb or ponderomotive) to materials in electric fields.

irradiated by an electromagnetic wave of frequency ω, as will be shown in detail, the refractive index n is given by the plasma frequency ω_p:

$$n^2 = 1 - \omega_P^2/\omega^2; \qquad \omega_P^2 = 4\pi e n_e/m, \tag{1.5}$$

where e is the charge, n_e is the density, and m is the mass of the electron. The (static) ponderomotive force of Kelvin acting at an electron fluid of density n_e, is expressed by the force density

$$f_K = m n_e \frac{d}{dt} v_e = \frac{n^2 - 1}{8\pi} \nabla \mathbf{E}^2, \tag{1.6}$$

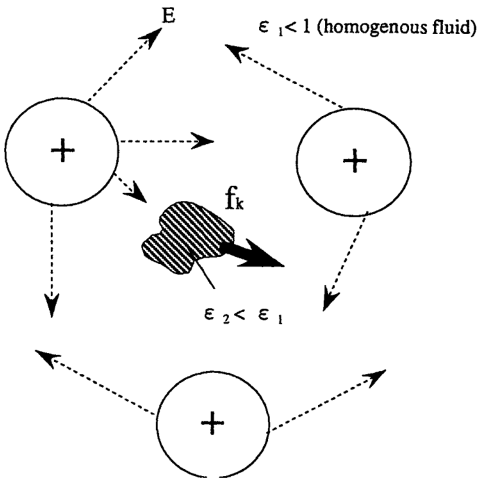

Figure 1.2 Dielectric material with a refractive index less than the surroundings within a minimum-*E* field being pushed to the minimum.

where v_e is the velocity of the electrons. Substituting the refractive index n from Eq. (1.5) one can cancel the electron density n_e and arrive at the equation of motion given by the force to one single electron:

$$f_{Ke} = m\frac{d}{dt}v_e = -\frac{e^2}{2\omega^2 m}\nabla\mathbf{E}^2. \tag{1.7}$$

This force, which can be expressed by the gradient of a scalar, is [like **E** in electrostatics, Eq. (1.4)] curl-free:

$$\nabla \times f_{Ke} = 0, \tag{1.8}$$

and the force can be expressed by the gradient of a mechanical potential:

$$f_{Ke} = -\nabla\phi; \qquad \phi = \frac{e^2}{2\omega^2 m}\mathbf{E}^2, \tag{1.9}$$

called the *ponderomotive potential*.

If the force in Eq. (1.7) is acting on an electron along a path between a point r_1 and a point r_2, the energy E gained or lost is independent of the path chosen for the integration (conservative force) and is given by

$$E = \int_{r_1}^{r_2} f_{Ke}\cdot dr = -\frac{e^2}{2\omega^2 m}[\mathbf{E}(r_2)^2 - \mathbf{E}(r_1)^2]. \tag{1.10}$$

If the ponderomotive potential is zero at r_2, $\phi(r_2) = 0$, the energy gained by the electron is given by the ponderomotive potential where it starts. Under stationary conditions, and if there is no emission or absorption of radiation (no Poynting flux), this can be applied to a cw-laser beam where an electron is generated in its center, e.g., by ionization from an atom. The electron is then emitted from the beam along the gradient of $\mathbf{E}^2$, i.e., in the radial direction of the beam, and it will gain the energy given by the ponderomotive potential [as measured first by Boreham et al. (1979)] where the laser intensity was high enough that the energy needed for the ionization could be neglected.

We shall see in the following that the calculation of the electron motion under these conditions is much more complex than a quiver drift. This usually arrives at the same global results as given by the ponderomotive potential.

It is therefore a point of caution to clarify whether the stationary conditions are fulfilled and the last term in Eq. (1.3) of the Kelvin ponderomotive force can be neglected or not. The questions of dissipation and of time-dependent interaction are then a further point of attention. It is interesting to note that the Helmholtz (1881) formulation of the ponderomotive force (Pavlov 1978) arrives for plasmas at the same expression as Kelvin's formulation. If the dielectric response given by the refractive index of solids is used, a discrepancy remains. This old controversy is in a similar way given for the question of what the correct relativistic description is for the propagation of light in media, the Abraham–Minkowski controversy. A transparent solution was possible for plasmas by reproducing Fresnel's formulas of reflectivity (Hora 1974): the photons behave half as Abraham predicted and half as Minkowski predicted [Klima and Petrzilka (1972); implicitly given by Hora (1969). See Hora (1991): Chapter 9.4], as was clarified by Nowak (1983). A very hesitantly formulated solution of the Abraham–Minkowski problem for the dielectric response of solid material by Sir Rudolf Peierls (Peierls 1976) could be confirmed and was basically founded on preceding measurements of the Schwarz–Hora effect (Hora et al. 2013).

It is indeed important to note that the ponderomotive potential acts on a free electron in the same way that an electric voltage U or the electrostatic potential ψ, Eq. (1.4), acts on an electron. The ponderomotive force is therefore similar, with its quadratic $\mathbf{E}^2$-gradient, as is the electric field $\mathbf{E}$ in the Coulomb force $\boldsymbol{f}_C$ acting on an electron:

$$\boldsymbol{f}_C = e\mathbf{E} = e\nabla\psi. \tag{1.11}$$

One only has to be aware that the electrodynamic force (apart from the electrostatic case) is more complex than the ponderomotive force, Eq. (1.7), and only the use of all components of Maxwell's stress tensor arrives at the correct solutions (Cicchitelli et al. 1990).

Historically, it is interesting to note that the ponderomotive force of the kind illustrated in Eq. (1.7) after its discussion in electrostatics—now without dielectric media—was first used for the time-dependent high-frequency case by Erich Weibel (1957, 1958) for the standing-wave field for microwaves (Fig. 1.3, however much more complicated in details, anticipating Fig. 7.2 calculated only after a number of iterations: Cicchitelli et al. 1990), followed by Gapunov and

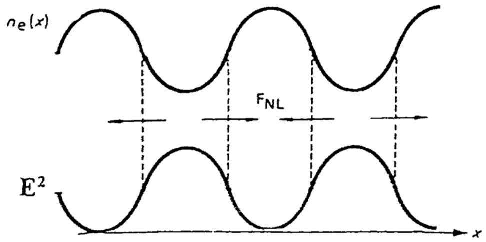

Figure 1.3 $\mathbf{E}^2$ for a standing-wave field, where the nonlinear (ponderomotive) force pushes the electrons into the nodes with a resulting electron density (dashed curve), as shown by Weibel (1957, 1958).

Miller (1958), and Boot et al. (1959). Weibel discovered that electrons will be driven into the nodes of the standing-wave field. Stationary laser beams interacting with electrons without dielectric effects were first discussed by Kibble (1966). The essential source of the dielectric effects on the electrodynamic forces at laser interaction with plasmas was first formulated by Hora et al. (1967) and generalized as a nonlinear force, where the macroscopic theory of the equation of motion in plasmas had to be modified by adding nonlinear terms in order to achieve momentum conservation (Hora 1969).

Despite all of the complications for the hydrodynamic theory, the result is rather easy to understand. If light in one dimension is moving into a plasma with monotonously increasing electron density n_e (Fig. 1.4), the averaged electric field squared $\overline{\mathbf{E}^2}$ of the laser light coming from vacuum with an amplitude $\mathbf{E}_v$ is being swelled up due to the refractive index n decreasing from the vacuum value to a minimum value at the critical density [although n in Eq. (1.5) is modified by absorption].

The resulting ponderomotive or nonlinear force $\mathbf{f}_{NL}$ is given by the negative gradient of $\mathbf{E}^2$ tearing the plasma into an ablative part and a compressive interior. The compression corresponds to an ordinary radiation

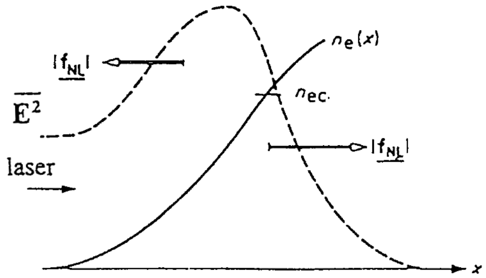

Figure 1.4 Electron density n_e depending on the plasma depth x with laser light incident from the vacuum side where n_e is zero. The averaged laser field $\overline{\mathbf{E}^2}$ grows over its vacuum value $\mathbf{E}_v$ due to the decreasing refractive index n to a maximum value near the critical electron density n_{ec}. The negative gradients of the $\mathbf{E}^2$-field are the nonlinear forces that drive the plasma corona towards the vacuum (ablation) and give a compression reaction to the plasma interior (Hora et al. 1967; Hora 1991).

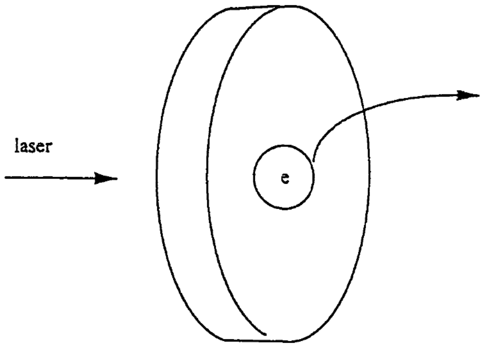

Figure 1.5 An 18-fs petawatt laser pulse focused to 50-μm diameter is like a pancake of photons. Hitting resting electrons will relativistically accelerate them as a free wave accelerator to GeV energies (Häuser et al. 1994; Barty 1996).

pressure, which, however, is increased by the swelling $1/n$. After detailed analysis, the Z-times ionized plasma ions emitted into the vacuum achieve an amount of energy with a value of

$$\varepsilon_{\mathrm{i}} = Z\frac{e^2}{2\omega^2 m}\mathbf{E}_{\mathrm{v}}^2\left[\frac{1}{|n|_{\min}} - 1\right], \tag{1.12}$$

which is exactly given by the change of the ponderomotive potential between the plasma interior and the vacuum. This linear separation of the ion energy was observed since the very first days of laser plasma interaction experiments and is characteristic of the action of the nonlinear (ponderomotive) force.* Any gas dynamic mechanism with thermal equilibrium of the ions cannot result in such a separation of the ions by their charge number Z.

The need to look into the details of these processes as explained in the following can be seen from an example where one calculates the interaction of initially resting electrons with "pancakes" of photons, e.g., 800-nm wavelength laser pulses of 18-fs duration (5.4 micrometer long) and of 50-μm focus diameter (Barty 1996), Fig. 1.5. The single electron interaction of the photon pancake of a few PW power will accelerate a single electron to energies in the GeV range (Häuser et al. 1994). It turns out that the maximum energies of the electrons are nearly as high as if the electron were accelerated within the ponderomotive potential. It is rather curious what the ponderomotive potential is able to reproduce, although it would not have been clear at the beginning how the thin pancake of photons would do this. Indeed, this

*This is the reason the processes treated in this book cannot simply be called "Radiation Pressure Acceleration (RPA)." A detailed account of dielectric response involved is given by Maxwell's stress tensor. An example is shown where all components of the tensor are needed to arrive at the correct nonlinear solution (Cicchitelli et al. 2000) to agree with the experiment. This turns out to be a principal reason in contrast to linear physics (Chapter 6.3).

all depends on further conditions (Häuser et al. 1994) to be explained. The critical point in this is that the purely electrostatic ponderomotive force of Kelvin in Eq. (1.1) has been extended to plasma for which, in some sense, no electrostatics exist and the high-frequency refractive index was used formally only to get a result from Eq. (1.1).

The ponderomotive force—generalized now for the nonelectrostatic properties of the plasma—is related to the Lorentz force, which is usually considered a typical relativistic result. We shall show how the Lorentz force can be derived from Maxwell's nonrelativistic theory, which indeed has the connection to Maxwell's stress tensor (see Section 3.2). Therefore, the plasma generalized ponderomotive force, the Lorentz force, the Maxwellian stress tensor, and the new aspects of plasma theory created by the very high laser intensity interaction with plasmas are interwoven. They are considered here in their full complexity in view of the basic background. The reference to Chris Barty (1996) was consequently following up Mourou's discovery of the initially mentioned generation of petawatt-picosecond laser pulses (Mourou 1994; Miley 1994; Perry et al. 1994), see Fig. 8.1 (Mourou 2011).

The ultrahigh acceleration as realized theoretically and numerically since 1978 (Hora 1981, Fig. 10.18a, b) as a nonthermal collective effect finally seen by the measurements of Sauerbrey (1996), is not only of importance for the fusion energy considered here but for particle acceleration in general. The accelerated plasma blocks with space charge neutrality represent ion acceleration with more than one million times higher ion current densities than as realized (Hora et al. 2002) from the experiments of Badziak et al. (1999) and even more for acceleration of electron or ion bursts with energies beyond GeV (Leemans et al. 2006) or aiming ion energies of PeV (Mourou et al. 2014). It is of interest to develop these generators of intense ion energy bursts of a few 100 MeV energy for very compact and economic facilities for the newly developed, highly successful noninvasive cancer treatment by hadron therapy (Banati et al. 2014). A summarizing overview of the development of the collective effect is in the appendix, and reviews are by Mourou et al. (2006), Krausz et al. (2009), Sprangle et al. (2007), and others. The computations presented here prefer hydrodynamic approaches, which are different from the wide field of using PIC (particle-in-cell) computations. As one example, it may be mentioned that the acceleration of blocks with hydrogen or helium nuclei up to the 70-MeV range arrived at comparable results with experiments using PIC (Gaillard et al. 2011), while hydrodynamics arrived in a straightforward way to the same agreement (Moustaizis et al. 2013).

The following description of the electrodynamic laser plasma interaction by the nonlinear (ponderomotive) forces is given in two steps. First, the macroscopic hydrodynamic plasma theory is discussed, and then the cases of single-particle motion are discussed.

In order to illustrate the deeper meaning of the nonlinear force and ponderomotion, a review of the basic framework of hydrodynamics and of electrodynamics will be given as would be appropriate for summarizing a graduate course where the student should have detailed knowledge of theoretical physics on hydrodynamics and on electrodynamics. We concentrate then on an elaboration and review of the main conceptual background of these fields, while the details of the mathematical derivations and proofs of the basic questions relevant to the nonlinear force and ponderomotion use an earlier reference (Hora 1991).

Coming back to Kelvin's historical discovery of ponderomotion in electrostatics, it should be realized that Coulomb discovered the electric forces between charges and that electric forces are also possible between uncharged neutral bodies with a dielectric response showing the quadratic (nonlinear) relation.

We shall mostly focus on the application of the dielectric and not immediately address related issues such as clean fusion, but we have to be aware what a wide field has to be covered. The upper end refers to accelerations at the surface of black holes with Hawking and Unruh radiation at intensities of pair production in vacuum (Stait-Gardner 2006; Eliezer et al. 2002: see Chapter 1.5), where electrons can no longer be fermions (Hora et al. 1961). The lower end of intensities is how the dielectric generated force at non-destructive intensities in living tissues can contribute to medical therapies.

The motivation for Kelvin's discovery was somewhat guided by observation. One could see similarities between diagrams of liquid flow and electric fields, or of static magnetic fields or the vector distribution of magnetic fields. We shall see that the relation between mechanical and electrical phenomena is linear only with the Coulomb force. In contrast to this linear relation, the general relation is basically nonlinear as seen from Maxwell's stress tensor including the here presented derivation of the Lorentz force (Section 3.2), which in some way opened a route to Einstein's theory of relativity. But it is even more interesting: in some generalized sense, the understanding of nonlinearity opens a new door for physics exploration as will be explained from discussions with Feynman (Section 6.3) and the position of Stephen Hawking that theoretical physics is not at the end, but nonlinearity is opening a door for a new dimension of knowledge.