Combining the fundamental principles of radiometry with a linear-systems description of scalar diffraction theory provides an intuitive understanding of a variety of diffraction phenomena usually thought to be beyond the realm of simple scalar theory. This includes the Wood's anomaly effect of redistributing energy from evanescent diffracted orders into propagating orders and certain nonintuitive surface scatter behavior at large incident angles.

In the February issue of this magazine, Bill Wolfe presented a review article on radiometry (see oemagazine, February 2001, page 33). We repeat here the definitions of three important radiometric quantities:

• Irradiance ($E$) is defined as the radiant power per unit area incident upon a surface.
• Radiant intensity ($I$) is power per unit solid angle radiated from a (point) source.
• Radiance ($L$) is defined as radiant power per unit solid angle per unit projected source area. Radiance is, in general, a function of position on the source and a function of the two angular variables $\theta_s$ and $\phi_s$ indicating the direction of the emitted radiation.

The fundamental quantity predicted by scalar diffraction theory is radiance, which greatly extends the range of parameters over which Fourier techniques can yield accurate predictions on wide-angle (non-paraxial) diffraction phenomena such as evanescent waves, diffracted intensity, and gratings.

We can develop a linear-systems formulation of non-paraxial scalar diffraction theory by normalizing the spatial variables by the wavelength of light:

$$\hat{x} = x / \lambda, \quad \hat{y} = y / \lambda, \quad \hat{z} = z / \lambda$$

and recognizing that the reciprocal variables in Fourier transform space are the direction cosines of the propagation vectors of the resulting angular spectrum of plane waves:

$$\alpha = \hat{x} / \hat{r}, \quad \beta = \hat{y} / \hat{r}, \quad \text{and} \quad \gamma = \hat{z} / \hat{r}$$

Theorists have shown that in direction cosine space only, wide-angle scalar diffraction phenomena are shift-invariant with respect to changes in incident angle. Furthermore, it is diffracted radiance (not intensity or irradiance) that is shift-invariant with respect to incident angle in direction cosine space. Hence for a diffracting aperture illuminated by a uniform amplitude plane wave with an incident angle specified by direction cosines ($\alpha_0 = 0, \beta_0$), the radiance $L$ is given by
\[ L(\alpha, \beta - \beta_o) = \gamma_o \frac{A_x}{A_\beta} |F\{U_o(\hat{x}, \hat{y} \hat{z}); 0\} \exp((2\pi \beta \hat{z})/\lambda)|^2 \]  

(3)

where \( U_o(\hat{x}, \hat{y} \hat{z}); 0\) is the complex amplitude distribution emerging from the diffracting aperture, \( A_\beta \) is the area of the diffracting aperture, and \( F\{ \} \) is symbolic notion for the Fourier transform operation. In other words, the fundamental quantity predicted by scalar diffraction theory is radiance.

As we mentioned earlier, this fact significantly extends the range of parameters over which simple Fourier techniques can be used to make accurate calculations concerning wide-angle (non-paraxial) diffraction phenomena. This revelation was perhaps so long in coming because there is no quantity corresponding to radiance in the standard electrical engineering terminology and nomenclature, and our Fourier optics texts have been written primarily by electrical engineers.

**evanescent waves**

In extreme cases, the above radiance distribution function extends beyond the unit circle in direction cosine space \((\alpha^2 + \beta^2 = 1)\), for example, when high spatial frequency content in the diffracting aperture yields very large diffracted angles, or when large incident angles shift a radiance distribution function to extend beyond the unit circle. In either case, the system generates evanescent waves. For energy to be conserved, the above equation for radiance must be renormalized. This is not done in a heuristic manner but is inherent to the scalar diffraction theory.

Rayleigh’s (Parseval’s) theorem from Fourier transform theory states that the integral overall space of the squared modulus of any function is equal to the integral over all space of the squared modulus of its Fourier transform, allowing us to use equation (3) to write

\[
\frac{P_T}{\lambda^2} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |L(\alpha, \beta - \beta_o) \exp((2\pi \beta \hat{z})/\lambda)|^2 \, d\alpha \, d\beta

= \frac{1}{\gamma_o} \frac{A_x}{A_\beta} \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} L(\alpha, \beta - \beta_o) \, d\alpha \right] \, d\beta

(4)

where \( P_T \) is the total radiant power emanating from the diffracting aperture, and

\[
L'(\alpha, \beta - \beta_o) = K L(\alpha, \beta - \beta_o)

(5)

is a normalized version of the original shifted radiance distribution function, \( L(\alpha, \beta - \beta_o) \), from equation (3). This normalization constant is given by the ratio of the integral of the on-axis radiance distribution function over infinite limits to the integral of the shifted radiance distribution function over the unit circle in direction cosine space.

\[
K = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} L(\alpha, \beta) \, d\alpha \, d\beta}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} L(\alpha, \beta - \beta_o) \, d\alpha \, d\beta}

(6)

Thus, for those cases in which evanescent waves exist, the diffracted radiance previously expressed in equation (3) must be normalized by the constant \( K \) so that energy will be conserved:

\[
L(\alpha, \beta - \beta_o) = K \frac{A_x}{A_\beta} |F\{U_o(\hat{x}, \hat{y} \hat{z}); 0\} \exp((2\pi \beta \hat{z})/\lambda)|^2

\text{for } \alpha^2 + \beta^2 \leq 1

(7)

\[
L(\alpha, \beta - \beta_o) = 0

\text{for } \alpha^2 + \beta^2 > 1

\]

This normalization constant, \( K \), differs from unity only if the radiance distribution function extends beyond the unit circle in direction cosine space (i.e., only if evanescent waves are produced). The well-known Wood’s anomalies that occur in diffraction grating efficiency measurements are entirely consistent with this predicted renormalization in the presence of evanescent waves.

**jack-of-all-trades, master of many**

It would be a challenge for anyone to pick Jim Harvey’s specialty. “My career has been all over the map,” he says. If you look closely, however, there is a common thread that runs throughout his career: analysis of diffraction effects, geometric aberrations, and surface scattering phenomena. “I tell my students these are the three fundamental mechanisms that degrade image quality,” Harvey says.

Of these, Harvey particularly focuses on surface-scattering phenomena. “I am so adamant about this area,” he continues, “because I can’t just go out and buy a code that does this. It isn’t sufficient to just do the scatter analysis. You have to incorporate it with image degradation.”

Harvey first started his crusade at the Ford Motor Company Scientific Research Staff where he became involved in applying optics and holography. After receiving his Ph.D., he went to work for Pratt and Whitney where he researched real-time wavefront sensing for adaptive optics. At Perkin Elmer (Danbury, CT), he worked primarily on modeling image quality for several of the NASA space telescopes, including the Chandra. At the Air Force Weapons Laboratory, he studied hydrogen energy beam and weapons laser technology.

Most recently, Harvey assumed a faculty position at the University of Central Florida’s Center for Research and Education in Optics and Lasers (CREOL; Orlando, FL). “When CREOL hired me 1990, people thought they were getting an optical designer,” says Harvey. “I’m not really, but I didn’t tell them that. Instead I just named my lab the ‘Optical Design and Image Analysis Laboratory.’”

In his lab, Harvey and associates are currently designing the x-ray telescopes that will be mounted on NOAA’s weather satellites to study sunspot and solar-flare activity, which can wreak havoc on the operation of space communications systems, global positioning systems, and even cell phones. “Our x-ray design will be launched next year,” says Harvey. “We feel like we’ve made a real difference in this program. In fact, I think we are making a significant contribution to scalar diffraction theory in general. Now, we just have to convince other people,” he says with a laugh. —Laurie Ann Toupin
**diffracted intensity**

Radiance is not a directly measurable quantity. In order to calculate radiant intensity to compare with experimental measurements, one need only apply Lambert's cosine law—multiply the radiance by \( \gamma = \cos \theta \)—and integrate over the source area \( A_s \):

\[
I(\alpha, \beta) = \int_{A_s} L(\alpha, \beta, x, y) \, dA_s
\]  

(8)

If the source is a uniformly illuminated diffracting aperture, we obtain diffracted intensity by multiplying equation 7 by \( A_s \gamma \):

\[
I(\alpha, \beta, \gamma) = K \gamma \int_{A_s} e^{i(2\pi \delta \phi)} \, dA_s
\]  

(9)

\[
I(\alpha, \beta, \gamma) = 0
\]

(10)

Although diffracted radiance may exhibit a discontinuity at the edge of the unit circle, the direction cosine \( \gamma \) in equation (9) ensures that diffracted intensity never exhibits such discontinuities (see figure 1).

**diffraction grating behavior**

The grating equation for an arbitrary obliquely incident beam can be written simply in direction cosine space. The diffracted orders will propagate along the surface of a cone and will strike an observation hemisphere in a cross-section that is not a great circle, but instead a latitude slice. The direction cosines are obtained by merely projecting the respective points on the hemisphere down onto the plane of the aperture and normalizing to a unit radius. For large angles of incidence and large diffracted angles, the various orders appear to be equally spaced, and lie on a straight line only in direction cosine space. Consider the case in which the location of the incident beam and the diffracted orders are displayed in direction cosine space for a grating whose grooves are parallel to the y-axis, with \( \psi = 90^\circ \), in which \( \psi \) is the angle between the grooves and the x-axis (see figure 2). When the plane of incidence is oblique to the grating grooves, we have the appropriate grating equation in direction cosine space:

\[
\alpha_m + \alpha_i = (m \lambda / d) \sin \psi
\]

\[
\beta_m + \beta_i = -(m \lambda / d) \cos \psi
\]

(10)

where \( d \) is the groove spacing and \( m \) is the diffracted order. When the plane of incidence is parallel to the grooves \( (\phi_o = 0) \), we obtain:

\[
\alpha_m = \sin \theta_m \cos \phi_o
\]

\[
\alpha_i = - \sin \theta_o \cos \phi_o
\]

\[
\beta_i = - \sin \phi_o
\]

(11)

and equation (10) reduces to the familiar form of the grating equation:

\[
\sin \theta_m + \sin \theta_i = m \lambda / d
\]

(12)

which is just the conventional grating equation for the case of plane diffraction only.

Note that the diffracted orders are always exactly spaced and lie in a straight line parallel to the \( \alpha \)-axis in direction cosine space. Those diffracted orders that lie inside the unit circle are real and propagate, whereas the diffracted orders that lie outside the unit circle are evanescent and thus do not propagate. The undiffracted zero order always lies diametrically opposite the origin of the \( \alpha-\beta \) coordinate system from the incident beam. As the incident angle is varied, the diffraction pattern (size, shape, separation, and orientation of diffracted orders) remains unchanged but merely shifts its position, maintaining the above relationship between the undiffracted order and the incident beam.

**surface scatter phenomena**

Rayleigh-Rice or Beckmann-Kirchoff theories are commonly used to predict surface scatter effects. Smooth surface or paraxial limitations are frequently imposed, however, and various published papers contain significant discrepancies between experimental measurements and theoretical predictions. Figure 2 illustrates three different nonintuitive effects that arise when comparing experimentally measured scattered intensity gathered by O’Donnell and Mendez from well-characterized surfaces with predictions from the classical Beckmann scattering theory.

The left plot reveals a persistent tendency for the measured data to be slightly narrower than the predicted scattering function, an
result in a narrower scattered intensity curve. For large angles of incidence, the scattered radiance function is substantially shifted in direction cosine space and abruptly truncated by the unit circle corresponding to scattering angles of 90°. When multiplied by Lambert’s cosine law, this discontinuity is replaced by a gradual decrease to zero at 9°. Renormalizing the volume to its original value causes the peak of the intensity curve to be substantially higher than the peak of the radiance curve. This process of multiplying by Lambert’s cosine law and renormalizing the volume to its original value also shifts the peak of the scattered intensity function away from the specular direction toward the surface normal. Finally, we obtain the excellent agreement between the scattered intensity predicted by this theory and the scattered intensity measurements of O’Donnell and Mender.

The integration of radiometry with a linear systems formulation of scalar diffraction theory provides new insight and understanding of nonparaxial diffraction phenomena. It also demonstrates that many shortcomings usually attributed to scalar diffraction theory are actually the result of an unnecessary paraxial limitation, and not inherent to the theory itself. Furthermore, the claim that radiance (not intensity or irradiance) is the fundamental quantity predicted by scalar diffraction theory is perhaps strengthened by the brightness theorem in geometrical optics, which states that the radiance of a lossless optical system is axially invariant throughout the system.

Once we recognize that the theoretical predictions represent scattered radiance and the experimental measurements are indeed scattered intensity, the discrepancies indicated in all three parts of figure 2 become intuitive. It is possible to achieve good agreement between theory and experiment by revisiting the scattering surface described in figure 2 using the approach described above. For small angles of incidence, multiplying the scattered radiance curve by Lambert’s cosine law will always.

References
For a complete list of the references associated with this article, please see the oemagazine website at www.oemagazine.com.